Draft version December 23, 2011
Preprint typeset using LaTeX style emulateapj v. 8/13/10

A POSSIBLE PHYSICAL CONNECTION BETWEEN HELIUM-RICH STELLAR POPULATIONS OF MASSIVE GLOBULAR CLUSTERS AND THE UV UPTURN OF GALACTIC SPHEROIDS

Kenji Bekki
ICRAR, M468, The University of Western Australia 35 Stirling Highway, Crawley Western Australia, 6009, Australia
Draft version December 23, 2011

ABSTRACT

We discuss a possible physical connection between helium-rich ($Y \geq 0.35$) stellar populations of massive globular clusters (GCs) and the ultraviolet (UV) upturn of galactic spheroids by using analytical and numerical models. In our model, all stars are initially formed as bound or unbound star clusters (SCs) formed from giant molecular clouds (GMCs) and the SCs can finally become GCs, open clusters, and field stars depending on physical properties of their host GMCs. An essential ingredient of the model is that helium-rich stars are formed almost purely from gas ejected from massive asymptotic giant branch (AGB) stars. The helium-rich star formation is assumed to occur within massive SCs if the masses of the progenitor GMCs are larger than a threshold mass ($M_{\rm thres}$). These massive SCs can finally become either massive GCs or helium-rich field stars depending on whether they are disintegrated or not. Using this model, we show that if the initial mass functions (IMFs) in galactic spheroids are mildly top-heavy, then the mass fractions of helium-rich main-sequence stars ($F_{\rm He}$) can be as large as ~ 0.1 for $M_{\rm thres} = 10^7 {\rm M}_{\odot}$. $F_{\rm He}$ is found to depend on IMFs and $M_{\rm thres}$ such that it can be larger for shallower IMFs and smaller $M_{\rm thres}$. The inner regions of galactic spheroids show larger $F_{\rm He}$ in almost all models. Based on these results, we suggest that if the UV upturn of elliptical galaxies is due to the larger fractions of helium-rich stars, then the origin can be closely associated with top-heavy IMFs in the galaxies.

Subject headings: galaxies: star clusters– globular clusters:general – stars:formation – galaxies:elliptical and lenticular, cD – ultraviolet:galaxies

1. INTRODUCTION

Recent observational and theoretical studies of the Galactic GCs have suggested that some of the massive GCs (ω Cen and NGC 2808) have significant fractions of helium-rich stars (e.g., Bedin et al. 2004; Norris 2004; Lee et al. 2005a; Piotto et al. 2005, 2007; Renzini 2008). The observed unusual level of helium enhancement ($\Delta Y/\Delta Z \approx 70$; Piotto et al. 2005) and sizable fractions of helium-rich stars in these GCs have driven many theoretical studies to make great efforts in understanding where and how helium-rich stars can be formed (e.g., Bekki & Norris 2006; D'Antona & Ventura 2007; D'Ercole et al. 2008). As discussed by Renzini (2008), helium-rich stars can be formed from gaseous ejecta from massive AGB stars with initial masses ($m_{\rm I}$) ranging from $3{\rm M}_{\odot}$ to $8{\rm M}_{\odot}$, though the ejecta should not be diluted by interstellar gas (ISM) to keep the original high helium abundances of the ejecta.

Code & Welch (1979) investigated the integrated light of seven elliptical and S0 galaxies for a wavelength range of 1550-4250 Å and found that some of them show an increase in energy at the shortest wavelengths. Since this discovery of the UV upturn, the origin of the UV upturn has been extensively discussed both in observational and theoretical studies (e.g., Bertola et al. 1980; Burstein et al. 1988; Greggio & Renzini 1990; Horch et al. 1992; Dorman et al. 1995; Brown et al. 1997; O'Connell 1999; Yi et al. 1997, 1998, 2005). Although there can be a number of stellar candidates responsible for the UV upturn in galaxies, one of the most promising ones is old horizontal-branch stars (e.g., Yi 2008 for a recent review). It is suggested that enhanced helium abundances can play an important role in the formation of hot stars responsible for the UV upturn (Yi 2008).

Brown et al. (2003) investigated the UV emission in eight early-type galaxies at $z = 0.33$ (a look-back time of 3.9 Gyr) and found that the UV emission in these galaxies is significantly weaker than it is in the current epoch. Observational studies by *Galaxy Evolution Explorer (GALEX)* investigated the UV properties of bright cluster galaxies (BCGs) in clusters at $z < 0.2$ and compared them with those of nearby giant elliptical galaxies (e.g., Lee et al. 2005b; Ree et al. 2007). Ree et al. (2007) concluded that the observed evolution of FUV $- V$ color with a model in which the dominant FUV source is hot horizontal-branch stars (e.g., Ree et al. 2007). Yi et al. (2011) investigated correlations between the strength of the UV upturn and global galactic properties (e.g., luminosity) in BCGs and did not find any remarkable correlations (see also Loubser & Sánchez-Blázquez 2011 for similar results). Yi et al. (2011) therefore concluded that the helium sedimentation scenario proposed by Peng & Nagai (2009) can not be supported by their observational results.

These observational and theoretical studies appear to suggest that there can be helium-rich stellar populations in diverse objects with dramatically different masses and sizes ranging from GCs to BCGs. Although it would be likely that different astrophysical objects with helium-rich stars have different origins, it would be possible that they can have a common origin. Previous theoretical studies, however, have not yet provided an unified picture for the origins of helium-rich stars in GCs, galactic bulges, and elliptical galaxies with the UV-upturn. It is therefore worthwhile to construct a model to discuss the

origins of helium-rich stars observed in GCs and galactic spheroids in a self-consistent manner.

The purpose of this paper is to present a new model which can provide a possible explanation for the origins of helium-rich stars with $Y \geq 0.35$ in GCs and galactic spheroids in a self-consistent manner. Based on the model, we mainly discuss what types of IMFs are required to explain the observed possible fraction of helium-rich stars in galactic spheroids with the UV upturn (e.g., Chung et al. 2011). The most important ingredient of the model is that helium-rich stars are formed almost purely from gas ejected from massive AGB stars with $3\mathrm{M}_{\odot} \leq m_{\mathrm{I}} \leq 8\mathrm{M}_{\odot}$ with no/little dilution of the gas with ISM. This formation of new stars from AGB ejecta have been included also in recent theoretical models of GC formation (e.g., D'Ercole et al. 2008).

In the present study, we assume that such formation of helium-rich stars almost directly from AGB ejecta can be possible only within SCs formed from massive GMCs. By considering that the vast majority of stars are observed to be initially formed as bound or unbound SCs (e.g., Lada & Lada 2003), we assume that all stars form as SCs from GMCs in the model. Some massive SCs can retain gaseous ejecta from AGB stars and consequently have new helium-rich stars formed form the ejecta, and the helium-rich stars can become field stars of their host galaxies when the SCs become disintegrated. Such massive SCs can finally become GCs or nuclear SC (or stellar galactic nuclei) depending on their birth places, if they are not destroyed by their host galaxies. The IMFs and GMCMFs are key parameter that can determine mass fractions of helium-rich stars in galaxies. By using this new model, we discuss the origin of physical properties of the Galactic GCs and the UV upturn in galactic bulges and elliptical galaxies.

Although it remains less clear how much fractions of stars should be helium-rich stars in galactic spheroids with the UV upturn (Yi 2008), Chung et al (2011) have recently suggested that if about 11% of stellar populations in galactic spheroids are helium-rich stars, then they can show the UV upturn for a given IMF. In their models, they considered that the major source of far-UV flux originates from metal-poor and helium-enhances hot horizontal branch stars. We therefore consider that if the fractions of helium-rich stars in galactic spheroids are less than ~ 0.1, the spheroids are unlikely to show the UV upturn in the present models. We investigate in what physical conditions (e.g., IMFs) the fractions of helium-rich stars can be above 0.1 for galactic spheroids to show clearly the UV upturn.

A possible physical link between helium-rich stellar populations and the UV upturn of galactic spheroids has been already pointed out by a number of authors (e.g., Yi 2008). Thus the main point of the present study is *not* to propose the importance of helium-rich stellar populations in the origin of the UV upturn *but to discuss how galactic spheroids can have significant fractions of helium-rich stellar populations.* It should be also stressed that the present model is idealized and less realistic in some points (e.g., no chemical evolution of galaxies): the model should be regarded as a first step toward better understanding the origin of helium-rich stars in GCs and galaxies. More sophisticated numerical simulations including SC formation processes in galaxies will need to be done in our future studies to address the UV upturn problem in a much more quantitative way.

The plan of the paper is as follows: in the next section, we describe a model which enables us to estimate (i) the mass fraction of helium-rich stars in a single GC and (ii) the mass fraction of helium-rich main-sequence (MS) stars in a galaxy for a given IMF and GMCMF. In §3, we present the results on the number/mass fraction of massive GCs with helium-rich stars in the Galaxy and the mass fractions of helium-rich MS stars in galaxies with different IMFs and GMCMFs In §4, we provide important implications of the present results in terms of the origin of helium-rich stars in GCs, the Galactic bulge, and elliptical galaxies. We also discuss the origin of the radial gradients of helium-rich stars in galactic spheroids in this section. We summarize our conclusions in §5.

2. THE MODEL

2.1. *A cluster disintegration scenario (CDS)*

2.1.1. *SCs as fundamental building blocks of galaxy formation*

We adopt a scenario in which (i) all stars in galaxies are formed as bound or unbound SCs from GMCs and (ii) helium-rich stars with $Y \geq 0.35$ can be formed from gas ejected from AGB stars only in massive SCs. Some SCs can become field stars in their host galaxies after disintegration and others can become massive GCs or nuclear star clusters (or stellar nuclei) in galaxies. Furthermore, if massive SCs with helium-rich stars are destroyed by their host galaxies, then the stars become helium-rich field populations in the galaxies. Therefore, formation efficiencies of massive SCs and IMFs in GCs and galaxies are key parameters which can determine their mass fractions of helium-rich stars. This scenario is referred to as "cluster disintegration scenario" (CDS) just for convenience in the present study. Since a number of possibly unfamiliar acronyms are used in the present study, their physical meanings are briefly summarized in Table 1 in order for readers to understand more clearly the present paper.

2.1.2. *Two stellar populations HNSs and HRSs with different Y*

In the CDS, stars formed directly from GMCs are assumed to have "normal" Y that is observed in HII regions of galaxies (e.g., Peimbert, Luridiana & Peimbert 2007) and predicted from canonical chemical evolution (i.e., $\Delta Y/\Delta Z \approx 2$) and they are referred to as "HNSs" (helium-normal stars). After these first generation of stars form, then the second generation of stars can form from gaseous ejecta of AGB stars among HNSs without dilution of the ejecta with ISM. These second generation of stars can show large degrees of helium enhancement (i.e., $\Delta Y/\Delta Z \geq 4$) which can not be expected in canonical chemical evolution models in which ISM and gaseous ejecta from stars are assumed to well mix and then form new stars. These second generation of stars are referred to as "HRSs" (helium-rich stars) just for convenience. As demonstrated by many recent theoretical studies (e.g., D'Antona et al. 2010), massive AGB stars can ejecta gas with enhanced helium abundances ($\Delta Y \approx 0.07$). Therefore it is quite reasonable to assume that HRSs can have high helium abundances. Figure 1

illustrates a whole picture for the formation histories of two different populations (HNSs and HRSs) from GMCs in the CDS.

Our recent numerical simulations on secondary star formation from AGB ejecta in SCs have shown that the ejecta can be converted efficiently into new stars only in massive SCs with masses ($m_{\rm sc}$) larger than $\sim 10^6 {\rm M}_{\odot}$ (Bekki 2010; 2011). This is because AGB ejecta can be accumulated in the deep potential wells of massive SCs to form high-density gaseous regions with the densities exceeding the threshold gas density ($\rho_{\rm thres}$) for star formation ($\rho_{\rm thres} \ge 10^4 - 10^5$ atoms cm^{-3}). We accordingly consider that AGB ejecta can be converted into new stars without the dilution with ISM if some physical conditions are met for SCs. We assume that (i) $m_{\rm sc}/m_{\rm gmc}$ is constant for all SCs and (ii) HRS formation is possible within SCs only if the masses ($m_{\rm gmc}$) of their host GMCs can exceeds a threshold mass ($M_{\rm thres}$). The introduction of $M_{\rm thres}$ is therefore consistent with recent numerical simulations by D'Ercole et al. (2008) and Bekki (2011).

2.1.3. *Y in HRSs*

It depends on chemical yields of AGB stars whether HRSs can have $Y \ge 0.37$ expected for blue main-sequence (MS) stars in ω Cen and NGC 2808 (e.g., Renzini 2008) and for the UV upturn in elliptical galaxies (e.g., Yi 2008). Although D'Antona et al. (2005) suggested that more massive AGB stars with $m_{\rm s} \ge 6{\rm M}_{\odot}$ can have $Y = 0.40$, AGB stars with lower $m_{\rm s}$ ($\le 5{\rm M}_{\odot}$) could have smaller Y (see Renzini 2008 for a more detailed discussion on this). It should be noted here that Portinari et al. (2010) suggested helium-rich populations in GCs to have $Y \approx 0.3$ rather than $Y \approx 0.4$. Considering these recent results, we adopt a model in which HRSs formed from gaseous ejecta of AGB stars with $3{\rm M}_{\odot} \le m_{\rm s} \le 8{\rm M}_{\odot}$ can become helium-rich populations of GCs and galactic spheroids.

As discussed in our previous study (Bekki & Norris 2006), HRSs with higher Y can be formed from ejecta of Type II supernovae, if the ejecta does not mix with ISM. We however does not discuss whether and how HRSs can be formed from the ejecta of Type II supernovae in the present study. Some observational results suggest that the helium-enrichment parameter $\Delta Y/\Delta Z$ can be larger than 4 (e.g., 5.3 ± 1.4 for local K-dwarf stars of the Galaxy; Gennaro et al. 2010). Therefore, it would be possible that even in canonical chemical evolution models, metal-rich stars with $Z \sim 0.03$ can have large Y (≥ 0.35) for the primordial helium content of $Y = 0.24$. Since we do not include chemical evolution of GCs and galaxies in the present study, we do not discuss the above possibility either.

2.2. *IMF*

We consider that HNSs and HRSs are formed in SCs with different IMFs. The adopted IMF in number is defined as follows:

$$\psi(m_{\rm I}) = m_{sc,0} {m_{\rm I}}^{-\alpha}, \tag{1}$$

where $m_{\rm I}$ is the initial mass of each individual star and the slope $\alpha = 2.35$ corresponds to the Salpeter IMF. The normalization factor $m_{sc,0}$ is a function of $m_{\rm sc}$, $m_{\rm l}$ (lower mass cut-off), and $m_{\rm u}$ (upper mass cut-off):

$$m_{sc,0} = \frac{m_{\rm sc} \times (2-\alpha)}{{m_{\rm u}}^{2-\alpha} - {m_{\rm l}}^{2-\alpha}}. \tag{2}$$

where $m_{\rm l}$ and $m_{\rm u}$ are set to be free parameters in the present study. The IMF slopes for HNSs and HRSs are denoted as α_1 and α_2 respectively, and $m_{\rm l,1}$ ($m_{\rm l,2}$) and $m_{\rm u,1}$ ($m_{\rm l,2}$) are for HNSs (HRSs).

Gaseous ejecta from AGB stars with masses ranging from $m_{\rm l,agb}$ to $m_{\rm u,agb}$ can contribute to the formation of HRSs and $m_{\rm l,AGB}$ and $m_{\rm u,agb}$ are fixed at $3{\rm M}_{\odot}$ and $8{\rm M}_{\odot}$, respectively. The total mass of AGB ejecta within a SC ($M_{\rm agb}$) is accordingly described as:

$$M_{\rm AGB} = \int_{m_{\rm l,agb}}^{m_{\rm u,agb}} m_{\rm ej} \psi(m) dm, \tag{3}$$

where $m_{\rm ej}$ describes the total gas mass ejected from an AGB star with initial mass $m_{\rm I}$ and final mass $m_{\rm F}$. We derive an analytic form of $m_{\rm ej}$ ($= m_{\rm I} - m_{\rm F}$) from the observational data by Weidemann (2000) by using the least-square fitting method, and find:

$$m_{\rm ej} = 0.916 m_{\rm I} - 0.444. \tag{4}$$

Using these equations, we estimate the mass fraction of AGB ejecta ($f_{\rm m,agb}$) in each individual SC for a given set of IMF parameters. $f_{\rm m,agb}$ is defined as follows:

$$f_{\rm m,agb} = \frac{M_{\rm agb}}{m_{\rm sc}}. \tag{5}$$

In order to discuss an increase of helium mass in a SC ($\Delta M_{\rm He}$) due to gas ejection of massive AGB stars, we adopt a formula used by Renzini (2008):

$$\Delta M_{\rm He} = 0.15(m_{\rm I} - 3){\rm M}_{\odot}. \tag{6}$$

We estimate the mass fraction of the fresh helium gas ($f_{\rm m,He}$) in each individual SC as follows:

$$f_{\rm m,He} = \frac{\int_{m_{\rm l,agb}}^{m_{\rm u,agb}} \Delta M_{\rm He} \psi(m) dm}{m_{\rm sc}}. \tag{7}$$

It is suggested that $f_{\rm m,He}$ can be 0.7 per cent for a Salpeter IMF and $m_{\rm l} = 0.5{\rm M}_{\odot}$ (Renzini 2008). If super AGB stars with $8 \le m_{\rm I} \le 10{\rm M}_{\odot}$ can also contribute to the production of helium-rich gas, then $f_{\rm m,He}$ can be 0.009 (Renzini 2008). These possibly small $f_{\rm m,He}$ made several authors to suggest that the original GCs are much more massive than the present ones (e.g., Bekki & Norris 2006).

2.3. *Mass fractions of HRSs in GCs*

We investigate the mass fractions of HRSs on the MS (simply referred to as "MS HRSs") in the present GCs ($f_{\rm He}$) In order to calculate the MS turn-off mass ($m_{\rm TO}$), we use the following formula (Renzini & Buzzoni 1986):

$$\ln m_{\rm TO}(t_{\rm s}) = 0.0558 (\log t_{\rm s})^2 - 1.338 \log t_{\rm s} + 7.764, \tag{8}$$

where $m_{\rm TO}$ is in solar units and time $t_{\rm s}$ in years. We assume that ages of GCs and galactic spheroids are 12 Gyr and thus $m_{\rm TO} = 0.885{\rm M}_{\odot}$. Although the present results can hardly depend on age differences of ~ 300 Myr (corresponding to the TO epoch of stars with $m_{\rm I} =$

$3\mathrm{M}_\odot$) between HNSs and HRSs, we consider that $m_{\rm TO} = 0.885\mathrm{M}_\odot$ for HNSs and $m_{\rm TO} = 0.889\mathrm{M}_\odot$ for HRSs.

The initial total mass of HRSs in a SC ($m_{\rm sc,2}$) is given as:

$$m_{\rm sc,2} = \epsilon_{\rm sf,2} M_{\rm AGB}, \tag{9}$$

where $\epsilon_{\rm sf,2}$ is the star formation efficiency for stars formed from AGB ejecta. For a strongly bound SC to be formed, the SC should not lose a significant fraction of gas left behind from star formation: $\epsilon_{\rm sf,2}$ needs to be larger than 0.5 (e.g., Hills 1980). We here adopt $\epsilon_2 = 1.0$ in the present study. The total mass of MS HNSs ($m_{\rm sc,mshns}$) in a SC is given as:

$$m_{\rm sc,mshns} = \int_{m_{\rm l,1}}^{m_{\rm TO}} m\psi(m)dm. \tag{10}$$

The total mass of MS HRSs ($m_{\rm sc,mshrs}$) in a SC as follows:

$$m_{\rm sc,mshrs} = \epsilon_{\rm sf,2} \int_{m_{\rm l,2}}^{m_{\rm TO}} m\psi(m)dm, \tag{11}$$

where the normalization factor of the IMF (ψ) is determined by $m_{\rm sc,2}$. Therefore, the mass fraction of MS HRSs in a SC is given as:

$$f_{\rm He} = \frac{m_{\rm sc,mshrs}}{m_{\rm sc,mshns} + m_{\rm sc,mshrs}}. \tag{12}$$

Thus $f_{\rm He}$ depends on α_1, α_2, $m_{\rm l,1}$, $m_{\rm u,1}$, $m_{\rm l,2}$, and $m_{\rm u,2}$.

2.4. *GMCMF*

In order to discuss the number/mass fraction of MGCs in the GC system (GCS) of the Galaxy and mass fractions of HRSs in galactic spheroids, we introduce GMCMFs. A galactic spheroid is initially composed of numerous GMCs from which unbound and bound SCs can be formed. We adopt the following GMCMF for GMCs:

$$\Psi(m_{\rm gmc}) = M_{g,0} {m_{\rm gmc}}^{-\beta}, \tag{13}$$

where β describes the slope of a GMCMF and is observed to be $1.6-1.7$ for galaxies in the Local Group (Rosolowski 2005). The normalization factor $M_{g,0}$ is a function of the total gas mass in a galactic spheroid ($M_{\rm g}$), $m_{\rm gmc,l}$ (lower mass cut-off), and $m_{\rm gmc,u}$ (upper mass cut-off):

$$M_{g,0} = \frac{M_{\rm g} \times (2-\beta)}{{m_{\rm gmc,u}}^{2-\beta} - {m_{\rm gmc,l}}^{2-\beta}}. \tag{14}$$

where $m_{\rm gmc,l}$ and $m_{\rm gmc,u}$ are set to be $10^3\mathrm{M}_\odot$ and $10^8\mathrm{M}_\odot$, respectively, in the present study.

We consider that β could be different in different regions of a galaxy and in different galaxies and thus investigate models with different β ($1.1 \le \beta \le 2.0$). We assume that star formation efficiencies ($\epsilon_{\rm sf,1}$) within host GMCs are constant, though they can be different between different GMCs. This is because we focus on mass fractions of HRSs in GCs and galactic spheroids and need to more clearly show their dependences on IMFs and GMCMFs. The present results do not depend so strongly on $m_{\rm gmc,l}$ and $m_{\rm gmc,u}$ for reasonable ranges of these two.

2.5. *Formation efficiency of bound SCs in GMCs*

Some SCs get disintegrated and consequently become field stars in their host galaxies and others survive from disintegration and finally become GCs. SCs with HRSs can finally become massive GCs (like ω Cen and NGC 2808), which are referred to "MGCs": other GCs with no HRSs are simply referred to as GCs. The total mass of SCs with HRSs formed in a galaxy is a key parameter which determines the total helium mass of the galaxy. Therefore, the formation efficiency of *bound* SCs ($\epsilon_{\rm bsc}$) in GMCs is one of the most important parameter in the present study. Here $\epsilon_{\rm bsc}$ is defined such that if $\epsilon_{\rm bsc} = 1$ (0.1), then each (one in ten) GMC can form a SC that is strongly bound so as to form stars from AGB ejecta.

Although it remains observationally unclear what a reasonable value is for $\epsilon_{\rm bsc}$, our previous simulations showed that formation efficiencies of GCs in starbursting galaxy mergers can be much higher in comparison with isolated disk galaxies owing to rather high pressure of ISM (Bekki et al. 2002). The simulations suggested that gas can be converted into strongly bound GCs rather than field stars (or weekly bound SCs) in major galaxy mergers. These results imply that if galactic spheroids are formed from major mergers, they can have rather high $\epsilon_{\rm bsc}$. We discuss $\epsilon_{\rm bsc}$ later in §4.

2.6. *Number and mass fractions of MGCs in the Galaxy*

We discuss number and mass fractions of MGCs ($F_{\rm n,mgc}$ and $F_{\rm m,mgc}$, respectively) can form "genuine GCs" that are observed to show O-Na anti-correlations and thus evidence for the presence of multiple stellar populations (e.g., Carretta et al. 2010) in the Galactic GCS. We consider that GMCs with $m_{\rm gmc} \ge 10^6$M (or $m_{\rm sc} \ge 10^5\mathrm{M}_\odot$ for $\epsilon = 0.1$) form "genuine GCs". The threshold GMC mass for the genuine GCs is denoted as $m_{\rm gmc,gc}$ for convenience. We estimate $F_{\rm n,mgc}$ as follows:

$$F_{\rm n,mgc} = \frac{\int_{M_{\rm thres}}^{m_{\rm gmc,u}} \epsilon_{\rm bsc}\Psi(m)dm}{N_{\rm gc}}, \tag{15}$$

where $N_{\rm gc}$ is the total number of genuine GCs and given as:

$$N_{\rm gc} = \int_{m_{\rm gmc,gc}}^{m_{\rm gmc,u}} \epsilon_{\rm bsc}\Psi(m)dm. \tag{16}$$

Likewise, $F_{\rm m,mgc}$ is estimated as follows:

$$F_{\rm m,mgc} = \frac{\int_{M_{\rm thres}}^{m_{\rm gmc,u}} \epsilon_{\rm sf,1}\epsilon_{\rm bsc} m\Psi(m)dm}{M_{\rm gc}}, \tag{17}$$

where $M_{\rm gc}$ is the total stellar mass of the genuine GCs and given as:

$$M_{\rm gc} = \int_{m_{\rm gmc,gc}}^{m_{\rm gmc,u}} \epsilon_{\rm sf,1}\epsilon_{\rm bsc} m\Psi(m)dm. \tag{18}$$

2.7. *Mass fractions of HRSs in galactic spheroids*

The total stellar mass of a galactic spheroid ($M_{\rm s}$) is estimated as follows:

$$M_{\rm s} = \int_{m_{\rm gmc,l}}^{m_{\rm gmc,u}} \epsilon_{\rm sf,1} m\Psi(m)dm. \tag{19}$$

The total mass of HRSs in the galactic spheroid ($M_{\rm s,hrs}$) is estimated as follows:

$$M_{\rm s,hrs} = \int_{M_{\rm thres}}^{m_{\rm gmc,u}} \epsilon_{\rm sf,1}\epsilon_{\rm sf,2}\epsilon_{\rm bsc} f_{\rm m,agb} m\Psi(m) dm. \quad (20)$$

In these equations, the term $\epsilon_{\rm bsc}$ is included only in the equation (20), because whether SCs are bound or unbound does not matter in estimating the total stellar mass $M_{\rm s}$. The mass fraction of HRSs in a galactic spheroid ($F_{\rm He,t}$) is therefore given as follows:

$$F_{\rm He,t} = \frac{M_{\rm s,hrs}}{M_{\rm s}}. \quad (21)$$

Although this $F_{\rm He,t}$ enables us to understand how much fraction of stars can be HRSs in a galactic spheroid, it is different from $F_{\rm He}$ which is more useful when the origin of galactic spheroids with HRSs are discussed.

The total mass of MS stars in a galactic spheroid ($M_{\rm s,ms}$) is estimated as follows:

$$M_{\rm s,ms} = \int_{m_{\rm gmc,l}}^{m_{\rm gmc,u}} \epsilon_{\rm sf,1} f_{\rm ms,1} m\Psi(m) dm, \quad (22)$$

where $f_{\rm ms,1}$ is the mass fraction of MS stars among all stars and depends on the IMF parameters of HNSs (e.g., α_1). The total mass of MS HRSs ($M_{\rm s,mshrs}$) is estimated as follows:

$$M_{\rm s,mshrs} = \int_{M_{\rm thres}}^{m_{\rm gmc,u}} \epsilon_{\rm sf,1}\epsilon_{\rm sf,2}\epsilon_{\rm bsc} f_{\rm ms,2} m\Psi(m) dm, \quad (23)$$

where $f_{\rm ms,2}$ is the mass fraction of MS HRSs among all stars formed as HRSs and determined by IMF parameters of HRSs. Therefore $F_{\rm He}$ is given as follows:

$$F_{\rm He} = \frac{M_{\rm s,mshrs}}{M_{\rm s,ms}}. \quad (24)$$

In the present study, $\epsilon_{\rm sf,1}$ is assumed to be independent of $m_{\rm gmc}$ and accordingly the present results on $F_{\rm He}$ do not depend on $\epsilon_{\rm sf,1}$. Since we focus on $F_{\rm He}$ in galactic spheroids, it is unimportant whether low-mass SCs are unbound (or bound) to become fields stars (open/globular clusters) after SC disintegration. The parameter $\epsilon_{\rm bsc}$ is therefore unimportant for GMCs with $m_{\rm gmc} \le M_{\rm thres}$. We assume that $\epsilon_{\rm bsc} = 1$ for GMCs that form MGCs in all models. It would be possible that $F_{\rm He}$ is significantly overestimated owing to the adopted assumption of $\epsilon_{\rm bsc} = 1$. We later discuss this point in §4. Thus free parameters are α_1, α_2, $m_{\rm l,1}$, $m_{\rm l,2}$, $m_{\rm u,1}$, $m_{\rm u,2}$, β, and $M_{\rm thres}$ in the present study. Table 2 and 3 briefly summarizes the physical meanings of these parameters and the definition of physical quantities investigated in the present study (e.g., $F_{\rm He}$), respectively, so that readers can understand more clearly the present results.

3. RESULTS

3.1. *GCs*

Figure 2 shows how $f_{\rm m,He}$ and $f_{\rm m,agb}$ depend on IMF parameters, α_1, $m_{\rm l,1}$, and $m_{\rm u,1}$. Clearly, $f_{\rm m,He}$ is rather small (~ 0.005) for a canonical IMF with $\alpha_1 = 2.35$, $m_{\rm l,1} = 0.1{\rm M}_{\odot}$, and $m_{\rm l,u} = 100{\rm M}_{\odot}$. This result means that only a small fraction of original stellar mass in a SC can be fresh helium gas that can be used for the formation of HRSs for a canonical IMF. However, $f_{\rm m,He}$ can be larger than 0.01 for top-heavy IMFs ($\alpha_1 < 2$) if $m_{\rm u}$ is smaller (e.g., 0.02 for α_1 and $m_{\rm u,1} = 8{\rm M}_{\odot}$). In extreme situations with $m_{\rm u,1} = 8{\rm M}_{\odot}$ (i.e., no Type II supernova), $f_{\rm m,He}$ can increase as the IMF slope decreases (i.e., shallower or more top-heavy IMF). Although top-heavy IMFs with $m_{\rm u,1} = 8{\rm M}_{\odot}$ would be highly unlikely for most GCs, these peculiar IMFs could be associated with the origins of some GCs with large fractions of HRSs. It is clear that $f_{\rm m,He}$ depends on $m_{\rm l,1}$ such that $f_{\rm m,He}$ is larger for larger $m_{\rm l,1}$.

The parameter dependences of $f_{\rm m,agb}$ on IMF parameters are essentially the same with those of $f_{\rm m,He}$. SCs with canonical IMFs with $\alpha_1 = 2.35$, $m_{\rm l,1} = 0.1{\rm m}_{\odot}$, and $m_{\rm u,1} = 100{\rm M}_{\odot}$ can have $f_{\rm m,agb} \sim 0.08$. However SCs with moderately top-heavy IMFs ($\alpha \sim 2$) can show $f_{\rm m,agb}$ as large as 0.2, if $m_{\rm l,1} = 0.5{\rm m}_{\odot}$ and $m_{\rm u,1}$ are smaller than $30{\rm M}_{\odot}$ (e.g., $f_{\rm m,agb} \sim 0.29$ for $m_{\rm l,1} = 0.5{\rm M}_{\odot}$ and for $m_{\rm u,1} = 8{\rm M}_{\odot}$). Given that these SCs can not lose significant fractions of their masses owing to the IMFs with rather small $m_{\rm u,1}$, they are highly likely to survive after supernova explosions so that secondary star formation can be possible within the SCs. If most of the AGB ejecta can be converted into HRSs within SCs, then the SCs can have significant fractions of HRSs. Thus the combination of moderately top-heavy IMFs and rather small $m_{\rm u,1}$ can significantly increase $f_{\rm m,agb}$, which suggests that SCs can contain significant fractions of HRSs without disintegration. The importance of such combination of $\alpha_1 \sim 2$ and low $m_{\rm u,1}$ in the formation of HRSs was not discussed in previous studies (e.g. Bekki & Norris 2006; Renzini 2008).

Figure 3 shows how $f_{\rm He}$ depends on α_1 for a given α_2 in SCs with $m_{\rm u,1} = 0.1{\rm M}_{\odot}$, $m_{\rm u,1} = 100{\rm M}_{\odot}$, and $\epsilon_{\rm sf,2} = 1$. Clearly, SCs with canonical IMFs with $\alpha_1 = 2.35$ and $\alpha_2 = 2.35$ have a small $f_{\rm He}$ (~ 0.08). SCs with more top-heavy IMFs in their HNSs (i.e., smaller α_1) show larger $f_{\rm He}$ for a given α_2. SCs with larger α_2 (more "bottom-heavy") show larger $f_{\rm He}$ for a given α_1. For SCs with canonical IMFs in their HRSs ($\alpha_2 = 2.35$) to have significant fractions of HRSs ($f_{\rm He} \sim 0.2$), they need to have top-heavy IMFs in HNSs ($\alpha_1 < 2$). These results do not depend so strongly on $m_{\rm l,2}$ and $m_{\rm u,2}$, though $f_{\rm He}$ can be larger for smaller $m_{\rm u,2}$. If SCs have too top-heavy IMFs (e.g., $\alpha_1 < 1.5$), then they can lose most of their mass (more than 50%) through Type II supernova explosions. These SCs are likely to become disintegrated before secondary star formation from AGB ejecta can proceed. Thus it is unlikely that SCs with large $f_{\rm He}$ (> 0.5) can be now observed as bound GCs.

SCs with moderately top-heavy IMFs ($\alpha_1 \sim 2$) can have significant fractions of HRSs ($f_{\rm He} \sim 0.2$), if these SCs can not be disintegrated due to mass loss through Type II supernova explosions. It should be however stressed that $\epsilon_{\rm sf,2} = 1$ and no mixing of AGB ejecta with ISM are assumed in these investigation. Furthermore, it is assumed that all of gaseous ejecta with AGB stars with different masses and lifetimes can be converted into new stars simultaneously. In real environments of GC formation, $\epsilon_{\rm sf,2}$ can be significantly smaller than 1 and AGB ejecta could be diluted by ISM. Also, massive stars among HRSs can explode as supernovae and consequently prevent gaseous ejecta from AGB stars with lower masses from being converted into new stars. Therefore $f_{\rm He}$ would be likely to be lower than those estimated

in the present study. It should be also stressed that if rather small $m_{\rm u,1}$ ($< 30{\rm M}_{\odot}$) is adopted, then $f_{\rm He}$ can be higher than those shown in Figure 3.

3.2. *The fraction of MGCs in the Galactic halo*

By assuming that all of the Galactic GCs were formed from GMCs, we here briefly discuss how much fraction of GCs were originally formed as MGCs with HRSs (like ω Cen and NGC 2808). In the present study, SCs formed from GMCs with $m_{\rm gmc} \ge M_{\rm thres}$ can efficiently convert their AGB ejecta into new stars without dilution by ISM so that they can have HRSs. Other less massive GC that evolve from SCs formed from GMCs with $m_{\rm gmc,gc} \le m_{\rm gmc} < M_{\rm thres}$ can finally become genuine GCs with no/few HRSs. SCs formed from GMCs with $m_{\rm gmc} \le m_{\rm gmc,gc}$ can not become genuine GCs in the present study. Given that the Galactic GCs with significant fractions of helium-rich populations (e.g., ω Cen) have total masses larger than $10^6{\rm M}_{\odot}$, we consider that $M_{\rm thres} = 10^7{\rm M}_{\odot}$ (for $\epsilon_{\rm sf,1} = 0.1$) is reasonable.

Figure 4 clearly shows that although the number fraction of MGCs is small ($F_{\rm n,mgc} \sim 0.16$) for a reasonable β (=1.7) and $M_{\rm thres} = 10^7{\rm M}_{\odot}$, the mass fraction ($F_{\rm m,mgc}$) is quite significant ($> 0.6$). Both $F_{\rm n,mgc}$ and $F_{\rm m,mgc}$ are larger for smaller β and smaller $M_{\rm thres}$ owing to the larger fractions of GMCs with $m_{\rm mgc} \ge M_{\rm thres}$. The essential reason for large $F_{\rm m,mgc}$ (> 0.5) in some models is that GMCMFs have slopes larger than -2 (i.e., $\beta < 2$). It should be stressed that $\epsilon_{\rm bsc}$ is assumed to be constant for all SCs in deriving these results. If host GMCs for MGCs have higher $\epsilon_{\rm bsc}$ than those for other GCs, then $F_{\rm n,mgc}$ and $F_{\rm m,mgc}$ can be even larger than those shown in Figure 4.

3.3. *Galactic spheroids*

Figure 5 shows that the mass fraction of HRSs ($F_{\rm He,t}$) in the model with $\beta = 1.7$ and $M_{\rm thres} = 10^7{\rm M}_{\odot}$ is rather small (~ 0.05) for canonical IMFs ($\alpha_1 = 2.35$). Since $\epsilon_{\rm bsc}$ is assumed to be 1 for all GMCs with $m_{\rm gmc} \ge M_{\rm thres}$ in the present study, $F_{\rm He,t}$ can be significantly smaller than 0.05 in real galaxies where $\epsilon_{\rm bsc}$ can be different in different star-forming regions and well less than 1 in some star-forming regions. These results mean that galactic spheroids are unlikely to have significant fractions of HRSs for canonical IMFs ($\alpha_1 = 2.35$). Clearly, $F_{\rm He,t}$ is larger for smaller β, because there are larger fractions of GMCs that have $m_{\rm gmc} \ge M_{\rm thres}$ and thus can host MGCs with HRSs in galactic spheroids. Also $F_{\rm He,t}$ is larger for smaller $M_{\rm thres}$ for a given β owing to larger fractions of GMCs with $m_{\rm gmc} \ge M_{\rm thres}$.

The mass fractions of MS HRSs ($F_{\rm He}$) can be significantly larger than $F_{\rm He,t}$ in galactic spheroids, if the mass fractions of MS HNSs are smaller owing to top-heavy IMFs. Figure 5 shows how $F_{\rm He}$ depends on α_1 for a given set of α_2, β, and $M_{\rm thres}$. As expected from Figure 4, $F_{\rm He}$ is rather small (~ 0.05) for canonical IMFs with $\alpha_1 = 2.35$ and $\alpha_2 = 2.35$. Irrespective of α_2, β, and $M_{\rm thres}$, IMFs of HNSs in galactic spheroids should be top-heavy ($\alpha_1 \le 2$) for the spheroids to have significant fractions of MS HRSs ($F_{\rm He} \ge 0.1$). It should be stressed, however, that the observationally suggested $F_{\rm He}$ (~ 0.1) for galactic spheroids with the UV upturn (Chung et al. 2011) can be explained by the models with mildly top-heavy IMFs of $\alpha_1 \sim 2$ that are not so exotic.

It is clear that $F_{\rm He}$ is larger for larger α_2 for a given set of α_1, β, and $M_{\rm thres}$. This is because larger numbers of low-mass HRSs ($m_{\rm I} < 3{\rm M}_{\odot}$) can be formed for larger α_2. Also, $F_{\rm He}$ can be larger for smaller β and smaller $M_{\rm thres}$, because larger numbers of GMCs with $m_{\rm mgc} \ge M_{\rm thres}$ can be formed. These results mean that whether galactic spheroids can have higher $F_{\rm He}$ (≥ 0.1) depends on physical properties of GMCs and physical processes of secondary star formation in MGCs. It should be stressed here that both $\epsilon_{\rm sf,2} = 1$ and $\epsilon_{\rm bsc} = 1$ are assumed in these estimation: top-heavy IMFs are required for galactic spheroids to have significant fractions of MS HRSs even in these maximum formation efficiencies of HRSs and MGCs.

4. DISCUSSION

4.1. *The lost GCs with helium-rich stars*

The Galactic GCs can sink into the inner region of the bulge owing to dynamical friction against the halo within ~ 13 Gyr, if their initial masses ($m_{\rm gc}$) is enough large (Binney & Tremaine 1987). By assuming that the Galaxy has a singular isothermal sphere, the time scale of dynamical friction for a GC ($t_{\rm df}$) can be estimated as follows:

$$t_{\rm df} = 2.3(\frac{\ln \Lambda}{10})^{-1}(\frac{r_{\rm i}}{2{\rm kpc}})^{2}(\frac{v_{\rm c}}{220{\rm pc}})(\frac{m_{\rm gc}}{10^7{\rm M}_{\odot}})^{-1}{\rm Gyr}, \tag{25}$$

where $\lg \Lambda$, $r_{\rm i}$, $v_{\rm c}$ are the Coulomb logarithm, the initial distance of the GC from the Galactic center, and the circular velocity of the Galaxy. In the above estimation, the reference value of $m_{\rm gc}$ was set to be a higher, because GCs can have significantly higher masses at their birth. The above equation means that initially massive GCs with HRSs have already sunk into the central region of the Galactic bulge owing to dynamical friction, if their $r_{\rm i}$ are less than 2 kpc. These GCs can not be observed as the Galactic halo GCs and contribute to the formation of helium-rich stars in the bulge.

In order to estimate how much fraction of massive GCs in the GCS of the Galaxy has been already lost in the bulge, we consider that the GCS has a spherical distribution with a density profile of $\rho(r) \propto r^{-3.5}$. This is consistent with that observed for the Galactic GCS (Djorgovski & Meylan 1994). The GCS is distributed within 35 kpc of the Galaxy and have a half-number radius of 5 kpc. We here investigate a threshold radius $R_{\rm thres}$ for which $t_{\rm df}$ can be smaller than 13 Gyr for a given $m_{\rm gc}$. GCs with $r_{\rm i} \le R_{\rm thres}$ can spiral into the center of bulge within 13 Gyr so that they can not be observed as the Galactic halo GCs. It is found that $\sim 49\%$ of GCs with $m_{\rm gc} = 10^7{\rm M}_{\odot}$ have been already lost for the above adopted GCS radial profile (i.e., $R_{\rm thres} = 4.7$ kpc). The fraction of these lost GCs ($F_{\rm lost}$) is smaller for smaller $m_{\rm gc}$: $F_{\rm lost}$ is 0.37 for $m_{\rm gc} = 5 \times 10^6{\rm M}_{\odot}$ and 0.06 for $m_{\rm gc} = 10^6{\rm M}_{\odot}$.

These results imply that what we can now observe as MGCs (e.g., ω Cen and NGC 2808) could be [51-63]% of original MGCs ($m_{\rm gc} \ge 5 \times 10^6{\rm M}_{\odot}$) formed in the Galactic halo. The lost MGCs would have been destroyed by the Galactic bulge and consequently their HRSs would have been dispersed into the inner bulge region: the stars could be now observed as HRSs in the bulge. This se-

lective loss of MGCs due to dynamical friction might well occur in galactic bulges and giant elliptical galaxies. Sohn et al. (2006) revealed a large number of GCs with strong UV light in M87 (see also Kaviraj et al. 2007). If these GCs contain large fractions of HRSs, then M87 could have already swallowed a fraction of these possible MGCs to add their HRSs to its main stellar spheroid.

4.2. *Top-heavy IMFs for the origin of the UV upturn in elliptical galaxies and BCGs*

The present study has demonstrated that if IMFs are top-heavy (i.e., $\alpha_1 < 2$), then significant fractions (~ 0.1) of MS stars in galactic spheroids can be HRSs. This is mainly because a larger amount of AGB ejecta with high helium abundances can be converted into new stars without dilution of the ejecta with ISM. This therefore implies that if the UV upturn is due largely to large fractions of MS HRSs within them, as observationally suggested (Chung et al. 2011), then galactic spheroids with mildly top-heavy IMFs are more likely to show the UV upturn. Then are the proposed top-heavy IMFs consistent with other observational properties of galactic spheroids such as their luminosity evolution and chemical abundances ?

A recent observation suggested that IMFs in massive galaxies at $0.02 \le z \le 0.83$ are significantly flatter (corresponding to $\alpha = 1.3$ in our model thus even shallower than the required slope of ~ -2) than the present-day value of the Galaxy and proposed a "bottom-light" IMF for massive galaxies at higher redshifts (van Dokkum 2008). This observational result is consistent with the present proposal and accordingly implies that if the observed massive galaxies in van Dokkum (2008) are progenitors for the present galactic spheroids, then some of them can show the UV upturn owing to larger $F_{\rm He}$. It should be stressed here that the observed fraction of spheroid populations with the UV upturn is small (e.g., Yi et al. 2011) and thus the observed *mean* properties of high redshift spheroids could be different from those of the present ones with the UV-upturn.

The required top-heavy IMFs are also consistent with the IMFs proposed for explaining the chemical properties of galactic spheroids (e.g., bulges), such as metallicity distribution functions (MDFs) and [α/Fe] (e.g., Matteucci & Brocato 1990; Nagashima et al. 2005; Ballero et al. 2007; Tsujimoto et al. 2010). Loewenstein (2006) showed that rapid star formation with top-heavy IMFs in massive galaxies are necessary to explain the observed Fe abundances of intra-cluster medium (ICM) which contains heavy metals from supernovae of massive early-type galaxies in cluster of galaxies. The top-heavy IMFs adopted in these previous studies suggest that the required IMF for explaining the UV-upturn observed in some spheroids is neither unreasonable nor unrealistic. Larson (1998) suggested that gas clouds with higher temperature in galaxies at high redshifts can be responsible for star formation with top-heavy IMFs. Thus the required top-heavy IMFs could be closely associated with high-redshift formation of the vast majority of stars in galactic spheroids.

It should be also stressed that if IMFs are too top-heavy (e.g., $\alpha_1 < 1.5$), then the SCs could become disintegrated before AGB ejecta can be recycled and converted into new stars. About 45% in mass can become stars with masses larger than $8{\rm M}_{\odot}$ (thus supernova) in SCs for $\alpha_1 = 1.95$, $m_{\rm l,1} = 0.1{\rm M}_{\odot}$, and $m_{\rm u,1} = 100{\rm M}_{\odot}$. These SCs can not become disintegrated and thus can continue secondary star formation, because more than 50% of their masses can still remain in SCs (e.g., Hills 1980). However, if $\alpha_1 = 1.5$, $m_{\rm l,1} = 0.1{\rm M}_{\odot}$, and $m_{\rm u,1} = 100{\rm M}_{\odot}$ then the mass fractions of supernovae in SCs are 0.74 so that SCs can lose most of their original masses. These SCs are highly likely to get disintegrated shortly after supernova explosions owing to their mass loss. Therefore, too top-heavy IMFs ($\alpha_1 < 1.5$) are not ideal for galactic spheroids to have helium-rich stellar populations.

Peng & Nagai (2009) proposed that sedimentation of helium in clusters of galaxies can be responsible for the formation of HRSs in BCGs. They also predicted that the UV flux strength is stronger in more massive, low-redshift, and dynamically relaxed BCGs. Their predictions, however, have been recently challenged by an observational study by Yi et al. (2011) which have found no correlation between the UV strength and rank/luminosities of BCGs and showed no clear difference in UV upturn fraction or strength in BCGs. The present study provides an alternative explanation for the origin of the BCGs with the UV upturn: BCGs show the UV upturn because IMFs at their formation were top-heavy ($\alpha_1 < 2$) and therefore they have larger $F_{\rm He}$. In the CDS with top-heavy IMFs, some BCGs can not show the UV upturn, because most stars are formed as SCs with canonical IMFs (whereas some can owing to top-heavy IMFs).

However, the origin of UV upturn in BCGs would not be so simple as the CDS with top-heavy IMFs envisages. Recent theoretical models have shown that hierarchical merging of smaller galaxies can play a vital role in the formation of BCGs (e.g., De Lucia & Blaizot 2007). The IMFs are observationally suggested to be different between faint and luminous galaxies (e.g., Hoversten & Glazebrook 2008). Therefore, if BCGs were formed by numerous mergers between galaxies with different luminosities in clusters, then not just IMFs but also merging histories (e.g., fractions of faint/luminous galaxies) can be key determinants for whether BCGs can show the UV upturn.

4.3. *Halo-spheroid connection*

Recently Martell et al. (2011) have revealed that about 3% of the Galactic halo stars can have depleted carbon and enhanced nitrogen abundances that are very similar to the chemical abundances observed for the "second-generation" of stars in GCs. Since these stars with characteristic chemical abundances can be formed from gaseous AGB ejecta of the "first generation of stars" (e.g., Bekki et al. 2007; D'Ercole et al. 2008), they suggested that (i) these stars originate from GCs and (ii) a minimum 17% of the present-day mass of the Galactic stellar halo was originally formed in GCs. Although it is difficult for observational studies to directly estimate helium abundances of stars in GCs, Bragaglia et al. (2010) have recently estimated the average enhancement in the helium mass fraction Y between the first and second generation (corresponding to HNSs and HRSs, respectively, in the present study) for 19 GCs with the Na-O anticorrelation. The estimated average enhancement is about $0.05 - 0.11$, which means that a significant fraction of

star in these GCs can have HRSs. If $\sim 10\%$ of all GC stars are HRSs, then about 1.7% of the halo stars can be regarded as HRSs.

This smaller number of 1.7% implies that the Galactic halo can not be identified as a galactic component with the UV upturn. Then what mechanism is responsible for the possible large difference in the mass fraction of HRSs ($F_{\rm He}$) between the Galactic stellar halo and galactic spheroids ? The present CDS suggests that most stars in the Galactic halo can originate from disintegration of low-mass SCs with canonical IMFs whereas those of galactic spheroids originate from stars initially in more massive SCs with top-heavy IMFs. Furthermore, as discussed in §4.1, the more massive SCs (or GCs) with HRSs can rapidly sink into the central regions of their hosts so that they can not contribute to the formation of HRSs in their halo regions: these more massive GCs can preferentially become the building blocks of the central spheroidal components in galaxies. Thus it is highly likely that $F_{\rm He}$ can be significantly different between halos and spheroids in galaxies.

4.4. *High formation efficiencies of bound massive SCs at the epoch of spheroidal formation*

It should be stressed here that $\epsilon_{\rm bsc} \sim 1$ has been so far assumed: GMCs with $m_{\rm gmc} \geq M_{\rm thres}$ need to almost always form bound SCs that can finally form helium-rich stars. If $\epsilon_{\ bsc} \sim 0.1$, then it is very hard for galactic spheroids to have $F_{\rm He} \sim 0.1$ even for very top-heavy IMFs (e.g., $\alpha_1 = 1$). Only spheroids with $\alpha_1 \sim 1$ and $m_{\rm u,1} \sim 8{\rm M}_{\odot}$ (i.e., peculiar top-heavy IMFs) can have $F_{\rm He}$ as large as 0.1 for $\epsilon_{\ bsc} \sim 0.1$. Photometric studies of super star clusters (SSCs) in a starbursting luminous infrared galaxy Arp 220 showed that the nuclear SSCs in Arp 220 contribute to $\sim 20\%$ of the total bolometric luminosity of Arc 220 (Shioya et al. 2001). Larsen & Richtler (2000) found that the formation efficiency of SCs can be higher in local regions with high star formation rate per unit area for disk galaxies. These observations imply that (i) a significant fraction of new stars are formed as SCs in a starburst or in high-density star-forming regions and thus (ii) galactic spheroids can have high cluster formation efficiencies if they were formed from massive starbursts in high-density environments.

However, owing to the lack of extensive observational studies on formation efficiencies of massive SCs in galaxies, it is currently very hard to discuss what a reasonable value of $\epsilon_{\ bsc}$ is for massive GMCs that can be progenitors for bound SCs with HRSs. If $\epsilon_{\ bsc}$ can be significantly larger than 0.1, then the present CDS is promising as the origin of the UV upturn in galactic spheroids, though top-heavy IMFs are still required. Thus, ultimately speaking, the origin of the UV upturn is closely related to the formation processes of MGCs from massive GMCs at the epoch of galaxy formation in the present CDS.

4.5. *Helium-rich populations in the Galactic bulge ?*

Nataf et al. (2011) revealed that the metal-rich stellar populations of the Galactic bulge can have $Y \sim 0.35$ based on the observational results on physical properties of the red giant branch bump (RGBB). They provided some important implications of the metal-rich HRSs in the bulge and suggested that galactic bulges in general can have HRSs like the bulge. The present results imply that if the Galactic bulge had a top-heavy IMF at its formation, then the present stellar populations can have a significant fraction (~ 0.1) of HRSs. Given that previous chemical evolution models of the Galactic bulge (e.g., Ballero et al. 2007; Tsujimoto et al. 2010) suggested a moderately top-heavy IMF ($\alpha \sim 2.05$) for the bulge, it is possible that the Galactic bulge can have a significant fraction of HRSs owing to the top-heavy IMF.

As suggested by Nataf et al. (2011), the dominant populations of the RGBB (with [Fe/H]> 0) in the bulge can have a rather high Y (~ 0.35): not just an $\sim 10\%$ of the bulge population needs to have $Y \sim 0.35$ to explain their observational results. If more than 30% of the metal-rich stellar populations have $Y \sim 0.35$, then the IMF slope (α) for the populations should be well less than 1.5 for $\beta = 1.7$ and $M_{\rm thres} = 10^7{\rm M}_{\odot}$. The required IMF is significantly more top-heavy than those suggested by chemical evolution studies (e.g., $\alpha \sim 2.05$; Tsujimoto et al. 2010) and thus would not explain the observed global properties of the bulge such as the metallicity distribution function of the bulge stars. However, if metal-rich stellar populations of the bulge has a very top-heavy IMF and if other populations have moderately top-heavy IMFs, then it would be possible that both the observed high fraction of metal-rich HRSs and global chemical properties in the bulge can be self-consistently explained. This possibility needs to be explored in our future studies based on detailed chemical evolution models.

4.6. *The origin of the correlation between the strength of UV upturn and Mg_2 index*

Recent observational studies of 48 nearby early-type galaxies by the SAURON project (Bureau et al. 2011) have confirmed the presence of a negative correlation between FUV - V color and Mg line strength originally proposed by Burstein et al. (1988). The present study provides the following possible explanation for the origin of this "Burstein relation" (i.e., a correlation of $\rm Mg_2$ with $(1550 - V)$ color). More massive elliptical galaxies can retain more efficiently the ejecta of supernovae so that they can finally have higher metallicities of their stars (e.g., Arimoto & Yoshii 1987). If more massive elliptical galaxies have shallower (i.e., more top-heavy) IMFs, then they can have large $F_{\rm He}$ and thus show the stronger UV upturn. Therefore more massive (or luminous) elliptical galaxies can have higher metallicities and thus higher $\rm Mg_2$ as well as the stronger UV upturn. Given that the observed positive correlations between luminosities, Mg indices, and velocity dispersions in elliptical galaxies (e.g., Faber & Jackon 1976; Bender et al. 1992), more massive (or luminous) elliptical galaxies can have higher $\rm Mg_2$, higher stellar velocity dispersion, and the stronger UV upturn. Thus, the dependence of IMF slopes on galactic masses/luminosities can be responsible for the origin of the Burstein relation.

Although Hoversten & Glazebrook (2008) revealed that galaxies significantly fainter than the Galaxy show steeper IMFs, it remain observationally unclear how IMF slopes depend on global galactic properties and formation environments of galaxies. Therefore it is too premature to conclude whether the CDS with IMF variation is

promising or not. As reviewed by Yi (2008), only a small fraction of elliptical galaxies show the UV upturn, which needs an explanation. If elliptical galaxies are formed from merging of smaller galaxies with different IMFs, then the origin of the UV upturn would not be so simple as the CDS explains.

The observed strong correlation between Mg_2 and $(1550 - V)$ color (Burstein et al. 1988) suggests that metallicities play a role in the formation of the correlation. Figure 1 in Burstein et al. (1988) also showed a large difference in $(1550 - V)$ colors (~ 2.5 mag) among elliptical with different Mg_2 ranging from ~ 0.20 to 0.36. It is unclear whether this large difference can be *quantitatively* explained by IMF variation alone, because the present study can not predict $(1550-V)$ colors as a function of $F_{\rm He}$. It would be possible that the combination of high metallicities and large $F_{\rm He}$ can make $(1550 - V)$ colors significantly bluer in elliptical galaxies. It is accordingly our future study to include chemical evolution and thereby to discuss the dependences of $F_{\rm He}$ on metallicities in elliptical galaxies.

4.7. *Radial gradients of helium-rich stars in galactic spheroids*

Carter et al. (2011) have recently investigated radial gradients of the FUV excess in 52 galaxies observed by *GALEX* and found that some of them show a positive gradient in the (*FUV* - *NUV*) color. They therefore suggested that the observed gradient can be due to a helium abundance gradient and furthermore that the presence of the gradient can place a strong limit on the importance of dry mergers in elliptical galaxy formation. In order to discuss the origin of the observed radial gradients of (*FUV* - *NUV*) color, we have constructed a toy N-body model in which a galactic spheroid is formed from merging of numerous SCs with the mass fraction of HRSs being a free parameter. The details of the model are given in the Appendix A.

Figure 7 shows time evolution of spatial distributions of HNSs and HRSs in a galactic spheroids composed initially of numerous SCs for the standard model. As SCs closely interact with one another in early evolution phase of the galactic spheroid, smaller and less massive SCs are destroyed by the tidal field of the host spheroid and by larger and more massive SCs. The stars (HNSs) initially in less massive SCs are dispersed during destruction of their host SCs and finally become field HNSs. On the other hand, massive SCs with $m_{\rm gmc} \geq M_{\rm thres}$ and thus with HRSs can not be destroyed efficiently during early multiple SC interaction. These massive SCs can sink into the inner region of the spheroid owing to dynamical friction against field stars (HNSs) and then interact dynamically will other massive SCs there. Consequently, the massive SCs can get disintegrated there and their HRSs can be dispersed to become field HRSs there. Since the inner region of the spheroid can be finally dominated by stars originally from HRSs of massive SCs, the inner region can have a larger fraction of HRSs within ~ 1 Gyr dynamical evolution.

Figure 8 shows how radial $F_{\rm He,t}$ gradients depend on models parameters, $M_{\rm thres}$ and γ, for a give β (=1.7). Clearly, all models shows negative gradients in the sense that inner regions of galactic spheroids have higher fractions of HRSs. For example, $F_{\rm He,t}$ can be ~ 0.08 within the central 200 pc in the standard model, though it is rather small (~ 0.01) in the outer region ($R \sim 1.8$ kpc). The parameter $M_{\rm thres}$ can control the central value of $F_{\rm He,t}$ such that $F_{\rm He,t}$ can be larger for smaller $M_{\rm thres}$. The $F_{\rm He,t}$ gradients can be smaller in models with $\gamma = 0$ in which there is no mass-size relation for SCs. The derived rather flat radial distribution is due to higher degrees of dynamical mixing of SCs with different mass in these models. Since $F_{\rm He}$ is proportional to $F_{\rm He,t}$ for a given set of IMFs, these results on $F_{\rm He,t}$ gradients can be true for $F_{\rm He}$. The present study accordingly demonstrates that galactic spheroids can show negative radial gradients of $F_{\rm He,t}$ and $F_{\rm He}$ and therefore negative gradients of the strength of the UV upturn (i.e., stronger in their inner regions). The present study thus can clearly explain the above observational result by Carter et al. (2011), if the observed color gradients are due largely to helium abundance gradients.

5. CONCLUSIONS

We have adopted a model in which (i) all stars are formed as bound or unbound SCs from GMCs and (ii) HRSs with $Y \geq 0.35$ can be formed from gas ejected from AGB stars only in massive SCs with $m_{\rm gmc} \geq M_{\rm thres}$. Some SCs can become field stars in their host galaxies after disintegration and others can become massive GCs or nuclear star clusters (or stellar nuclei) in galaxies. If massive SCs with HRSs are destroyed by their host galaxies, then HRSs become helium-rich field stars in the galaxies. We have investigated mass fractions of MS HRSs ($f_{\rm He}$ and $F_{\rm He}$) in GCs and galactic spheroids and their dependences on IMFs, $M_{\rm thres}$, and GMCMFs based on the CDS ("cluster disintegration scenario"). We have also investigated the radial gradients of $F_{\rm He}$ in galaxies with mass-size relations of GMCs by using N-body simulations on dynamical evolution of multiple cluster systems. We summarize our principle results as follows.

(1) The mass ratios ($f_{\rm m,agb}$) of gaseous ejecta from massive AGB stars with $3{\rm M}_{\odot} \leq m_{\rm I} \leq 8{\rm M}_{\odot}$ to initial total masses of SCs are typically ~ 0.08 for a canonical IMF ($\alpha_1 = 2.35$) with the reasonable lower and upper cut-off masses. If these gas can be efficiently converted into new stars with enhanced helium abundances (i.e., HRSs) in SCs, then the mass fractions of MS HRSs among all MS stars ($f_{\rm He}$) are ~ 0.08 for canonical IMFs and a SC age of 12 Gyr. Only if original SCs have top-heavy IMFs (e.g., $\alpha_1 \sim 1.5$), then $f_{\rm He}$ can be larger than 0.3 for $\alpha_2 \geq 1.85$.

(2) The initial number and mass fractions of massive GCs (MGCs) with HRSs ($F_{\rm n,mgc}$ and $F_{\rm m,ngc}$, respectively) among the Galactic GCs formed from GMCs with $m_{\rm gmc} \geq 10^6{\rm M}_{\odot}$ is ~ 0.16 and 0.66, respectively, for $M_{\rm th} = 10^7{\rm M}_{\odot}$ and $\beta = 1.7$. Both $F_{\rm n,mgc}$ and $F_{\rm m,mgc}$ are lager for smaller $M_{\rm th}$ and smaller β. MGCs with $m_{\rm gc} = 5 \times 10^6{\rm M}_{\odot}$ can sink into the Galactic center owing to dynamical friction to disappear from the halo region within 13 Gyr, if the initial radii from the Galactic center are less than 3.4 kpc. About 37% (49%) of the Galactic GCs with $m_{\rm gc} \geq 5 \times 10^6{\rm M}_{\odot}$ ($m_{\rm gc} \geq 10^7{\rm M}_{\odot}$) have already been lost from the halo owing to efficient dynamical friction of these massive GCs. Thus, the

Galactic halo initially could have a larger fraction of MGCs.

(3) The mass fractions of HRSs among all stars ($F_{\rm He,t}$) in galactic spheroids are ~ 0.05 for $M_{\rm thres} = 10^7{\rm M}_{\odot}$, $\beta = 1.7$, $f_{\rm m,agb} = 0.1$, and $\epsilon_{\rm bsc} = 1$. $F_{\rm He,t}$ of galactic spheroids are larger for smaller $M_{\rm th}$ and smaller β for a given IMF. The mass fractions of MS HRSs among all MS stars ($F_{\rm He}$) in galactic spheroids are 0.06 for canonical IMFs ($\alpha_1 = \alpha_2 = 2.35$), $M_{\rm th} = 10^7{\rm M}_{\odot}$, $\beta = 1.7$, and a galaxy age of ~ 12 Gyr. $F_{\rm He}$ in galactic spheroids can be larger for smaller $M_{\rm th}$ and smaller β for a given IMF, and it can be significant (> 0.1), if original SCs (i.e., building blocks of the spheroids) have top-heavy IMF (e.g. $\alpha_1 < 2$). These results suggest that if the observed UV upturn in bright elliptical galaxies is due to the larger fractions ($\sim 10\%$) of HRSs within them, the IMFs need to be top-heavy. If $\epsilon_{\rm bsc}$ is as small as ~ 0.1, then bright elliptical galaxies are unlikely to show the UV upturn owing to the small fractions of HRSs.

(4) The Galactic bulge can have a larger $F_{\rm He}$ if it had a top-heavy IMF at its formation in the CDS. Given a number of suggestions on the top-heavy IMF of the bulge by chemical evolution models of the bulge, the observed possible presence of helium-rich stars with $Y \sim 0.35$ in the bulge (e.g., Nataf et al. 2011) can be understood in the context of the CDS. HRSs in MGCs that were initially located in the Galactic halo and had sunk into the bulge can be currently observed as field HRSs in the bulge, though the contribution of such stars to the entire helium-rich population is rather minor. It is doubtlessly worthwhile for observational studies to investigate whether or not the bulge has a larger fraction of HRSs in the inner regions.

(5) If $M_{\rm th}$ and β are constant in elliptical galaxies with different physical properties, then elliptical galaxies with shallower (or more top-heavy) IMFs can show larger fraction of HRSs. Therefore, if more massive/luminous elliptical galaxies have shallower IMFs, then they can have larger fraction of HRSs. More massive elliptical galaxies can retain more efficiently the ejecta of supernovae so that they can finally have higher metallicities of their stars (e.g., Arimoto & Yoshii 1987). Therefore, if the Burstein relation (a correlation between UV flux and Mg index; Burstein et al. 1988) in bright elliptical galaxies is due to the correlation between $F_{\rm He}$ and Mg indices, then the relation can be understood in terms of shallower IMF slopes (α_1) in more massive/luminous elliptical galaxies.

(6) Galactic spheroids can show negative radial gradients of $F_{\rm He,t}$ and $F_{\rm He}$ (i.e., higher in inner parts) for a reasonable set of model parameters. This is mainly because massive SCs with HRSs can sink rapidly into the inner regions of galaxies owing to dynamical friction and disintegrate there. The HRSs initially in the SCs can be dispersed to become field HRSs there after disintegration of the host SCs. Therefore they can show the stronger UV upturn in their inner regions, if $F_{\rm He}$ can determine the strengths of the UV upturn. The observed spatial distribution of the FUV excess in elliptical galaxies can be understood in the context of formation and evolution of MGCs with HRSs. The central regions of galactic spheroids can be composed of two different stellar populations with normal helium abundances and enhanced ones (i.e., HNSs and HRSs).

Thus, an advantage of the CDS is that both (i) the absence or presence of the UV upturn in galactic spheroids and (ii) correlations between the strength of the UV upturn and galactic properties (e.g., $\rm Mg_2$ index) can be discussed in the context of IMF slopes of galaxies in a self-consistent manner. A disadvantage of the CDS is that it is unclear whether and in what physical conditions $\epsilon_{\rm bsc}$ can be ~ 1 for massive GMCs in galaxies. The formation of HRSs is possible, if helium-rich gas from AGB stars is converted into new stars without dilution of the gas by ISM with normal helium abundances. In the CDS, the inner regions of massive SCs are assumed to be the production sites of HRSs. It would be possible that AGB ejecta can be converted into new cold gas clouds outside SCs and consequently into new stars in deep potential wells of galactic spheroids, if there is no/little cold gas with normal helium abundances there. We need to investigate where and how AGB ejecta from field stars and SCs can be converted into new stars in galaxies using a more sophisticated chemodynamical simulation in our future studies.

6. ACKNOWLEDGMENT

I am grateful to the anonymous referee for valuable comments which contribute to improve the present paper. KB acknowledge the financial support of the Australian Research Council throughout the course of this work.

APPENDIX

N-BODY MODELS FOR MULTIPLE SC EVOLUTION

We investigate dynamical evolution of a galactic spheroid composed of numerous SCs using collisionless N-body simulations based on an idealized model for galactic spheroids. The main aim of this numerical investigation is to illustrate the radial gradients of the mass fractions of HRSs ($F_{\rm He,t}(R)$) in the spheroids. This idealized model is used only in the present preliminary study for the origin of helium-rich stars in galactic spheroids, and more sophisticated models including chemical evolution due to gaseous ejection from supernovae and AGB stars will be used in our future studies. However, we consider that this "toy" model enables us to grasp some essential ingredients of the formation and evolution of radial $F_{\rm He,t}$ gradients in galactic spheroids.

A galactic spheroid has an initial total mass $M_{\rm gal}$ and a size $R_{\rm gal}$ and composed of SCs with the total number of $N_{\rm sc,t}$. The radial distribution of SCs are described by a Plummber profile (e.g., Binney & Tremaine 1987) with the scale-length of $0.2R_{\rm gal}$. Each SC with a mass $m_{\rm sc}$ and a size $r_{\rm sc}$ is composed of many stars and has a Plummber density

profile with the scale-length of $0.2r_{\rm sc}$. A galactic spheroid is assumed to be initially in dynamical equilibrium, so the velocities of each SCs is given according to the velocity dispersion profile of the adopted Plummer model.

A SC is composed of HNSs and HRSs and these two populations have different initial distributions within the SC. Recent numerical simulations have shown that secondary star formation from AGB ejecta can proceed mostly in the central regions of MGCs (e.g., D'Ercole et al. 2008; Bekki 2010, 2011). Guided by these results, HNSs and HRSs are located at $0.1r_{\rm sc} \le R \le r_{\rm sc}$ and $R < 0.1r_{\rm sc}$, respectively. The mass fraction of HRSs are calculated for a given β and $M_{\rm thres}$. The total particle number in a SC is proportional to $m_{\rm sc}$ so that masses of stellar particles can be all the same.

A progenitor GMC for a bound/unbound SC has a mass $m_{\rm gmc}$ and a size $r_{\rm gmc}$. If we use the observed relation between mass densities and sizes of GMCs discovered by Larson (1981) and the observed typical mass and size of GMCs in the Galaxy (e.g., Solomon et al. 1979), then $m_{\rm gmc} - r_{\rm gmc}$ relation can be described as follows:

$$r_{\rm gmc} = 40 \times (\frac{m_{\rm gmc}}{5 \times 10^5 {\rm M}_{\odot}})^{\gamma} {\rm pc}, \tag{A1}$$

where $\gamma \sim 0.5$. We investigate models with different γ, because initial SCs can have mass-size relations different from those of GMCs. We consider that when SCs are formed, SCs have a mass-size relation similar to the above one and thus assume that $r_{\rm sc}$ can be the same as $r_{\rm gmc}$ described above for a given $m_{\rm gmc}$.

We mainly present the results of the "standard" dynamical models in which $M_{\rm gal} = 10^9 {\rm M}_{\odot}$, $R_{\rm gal} = 2$ kpc, $\beta = 1.7$, $M_{\rm thres} = 10^7 {\rm M}_{\odot}$ $m_{\rm gmc,l} = 10^4 {\rm M}_{\odot}$, $m_{\rm gmc,u} = 10^8 {\rm M}_{\odot}$, and $\gamma = 0.5$, though we investigate models with different parameters. In order to perform numerical simulations, we use the latest version of GRAPE (GRavity PipE, GRAPE-DR), which is the special-purpose computer for gravitational dynamics (Sugimoto et al. 1990). The total particle number used in a simulation is ~ 200000, which we consider to be reasonable for this preliminary investigation. The gravitational softening length is set to be 20 pc for all models. We focus only on the final radial gradients of $F_{\rm He,t}$ in galactic spheroids formed from dynamical evolution of numerous SCs.

REFERENCES

Arimoto, N., & Yoshii, Y. 1987, A&A, 173, 23
Ballero, S. K., Matteucci, F., Origlia, L., & Rich, R. M. 2007, A&A, 467, 123
Bedin, L. R., Piotto, G., Anderson, J., Cassisi, S., King, I. R., Momany, Y., & Carraro, G. 2004, ApJ, 605, L125
Bekki, K. 2010, ApJ, 724, L99
Bekki, K. 2011, MNRAS, 412, 2241
Bekki, K., Forbes, D. A., Beasley, M. A., & Couch, W. J. 2002, MNRAS, 335, 1176
Bekki, K., & Norris, J. E. 2006, ApJ, 637, L109
Bekki, K., Campbell, S. W., Lattanzio, J. C., & Norris, J. E. 2007, MNRAS, 377, 335
Bender, R., Burstein, D., & Faber, S. M. 1992, ApJ, 399, 462
Bertola, F., Capaccioli, M., Holm, A. V., & Oke, J. B. 1980, ApJ, 237, L65
Binney, J., & Tremaine, S. 1987 in Galactic Dynamics.
Bragaglia, A., Carretta, E., Gratton, R., D'Orazi, V., Cassisi, S., & Lucatello, S. 2010, A&A, 519, 60
Brown, T. M., Ferguson, H. C., Davidsen, A. F., & Dorman, B. 1997, ApJ, 482, 685
Brown, T. M., Ferguson, H. C., Smith, Ed,, Bowers, C. W., Kimble, R. A., Renzini, A., & Rich, R. M. 2003, ApJ, 584, L69
Bureau, M., et al. 2011, MNRAS, 414, 1887
Burstein, D., Bertola, F., Buson, L. M., Faber, S. M., & Lauer, T. R. 1988, ApJ, 328, 440
Carretta, E., Bragaglia, A., Gratton, R. G., Recio-Blanco, A., Lucatello, S., D'Orazi, V., & Cassisi, S. 2010, A&A, 516, 55
Carter, D., Pass, S., Kennedy, J., Karick, A. M., & Smith, R. J. 2011, MNRAS, 414, 3410
Chung, C., Yoon, S. J., & Lee, Y. W. 2011, ApJ, 740, L45
Code, A. D., & Welch, G. A. 1979, ApJ, 228, 95
D'Antona, F., Bellazzini, M., Caloi, V., Pecci, F. F., Galleti, S., & Rood, R. T. 2005, ApJ, 631, 868
D'Antona, F., & Ventura, P. 2007, MNRAS, 379, 1431
D'Antona, F., Ventura, P., Caloi, V., D'Ercole, A., Vesperini, E., Carini, R., & Di Criscienzo, M. 2010, ApJ, 715, 63
De Lucia, G., & Blaizot, J. 2007, MNRAS, 2007, 374, 2
D'Ercole, A., Vesperini, E., D'Antona, F., McMillan, S. L. W., & Recchi, S. 2008, MNRAS, 391, 825
Djorgovski, S., & Meylan, G. 1994, AJ, 108, 1292
Dorman, B., O'Connell, R. W., & Rood, R. T. 1995, ApJ, 442, 105
Faber, S. M., & Jackson, R. E. 1976, ApJ, 204, 668
Gennaro, M., Prada M., P. G., & Degl'Innocenti, S. 2010, A&A, 518, 13
Greggio, L., & Renzini, A. 1990, ApJ, 364, 35
Hills, J. G. 1980, ApJ, 235, 986
Horch, E., Demarque, P., & Pinsonneault, M. 1992, ApJ, L388
Hoversten, E. A., & Glazebrook, K. 2008, ApJ, 675, 163
Kaviraj, S., Sohn, S. T., O'Connell, R. W., Yoon, S.-J., Lee, Y. W., & Yi, S. K. 2007, MNRAS, 377, 987
Lada, C. J., & Lada, E. A. 2003, ARA&A, 41, 57
Larsen, S. S., & Richtler, T. 2000, A&A, 354, 836
Larson, R. B. 1981, MNRAS, 194, 809
Larson, R. B. 1998, MNRAS, 301, 569
Lee, Y. W., et al. 2005a, ApJ, 621, L57
Lee, Y. W., et al. 2005b, ApJ, 619, L103
Loubser, S. I., & Sánchez-Blázquez, P. 2011, MNRAS, 410, 2679
Loewenstein, M. 2006, ApJ, 648, 230
Martell, S. L., Smolinski, J. P., Beers, T. C., & Grebel, E. K. 2011, A&A, 534, 136
Matteucci, F., & Brocato, E. 1990, ApJ, 365, 539
Nagashima, M., Lacey, C. G., Okamoto, T., Baugh, C. M., Frenk, C. S., & Cole, S. 2005, MNRAS, 363, L31
Nataf, D. M., Udalski, A., Gould, A., & Pinsonneault, M. H. 2011, ApJ, 730, 118
Norris, J. E. 2004, ApJ, 612, 25
O'Connell, R. W. 1999, ARA&A, 37, 603
Peimbert, M., Luridiana, V., & Peimbert, A. 2007, ApJ, 666, 636
Peng, F., & Nagai, D. 2009, ApJ, 705, L58
Piotto, G. et al. 2005, ApJ, 621, 777
Piotto, G. et al. 2007, ApJ, 661, L53
Portinari, L., Casagrande, L., & Flynn, C. 2010, MNRAS, 406, 1570
Ree, C. H., et al. 2007, ApJS, 173, 607
Renzini, A. 2008, MNRAS, 391, 354
Renzini, A., & Buzzoni, A. 1986, in Spectral evolution of galaxies, (Dordrecht, D. Reidel Publishing Co.), p.195
Rey, S.-C., et al. 2007, ApJS, 173, 643
Rosolowsky, E. 2005, PASP, 117, 1403
Shioya, Y., Taniguchi, Y., & Trentham, N. 2001, MNRAS, 321, 11
Sohn, S. T., O'Connell, R. W., Kundu, A., Landsman, W. B., Burstein, D., Bohlin, R. C., Frogel, J. A., & Rose, J. A. 2006, AJ, 131, 866
Solomon, P. M., Sanders, D. B., & Scoville, N. Z. 1979, in Proc. IAU Symp. 84, The Large-scale Characteristics of the Galaxy, ed. W. B. Burton (Dordrecht: Reidel), 35
Sugimoto, D., Chikada, Y., Makino, J., Ito, T., Ebisuzaki, T., & Umemura, M. 1990, Nature, 345, 33

Tsujimoto T., Bland-Hawthorn J., & Freeman K. C. 2010, PASJ, 62, 447
van Dokkum, P. G. 2008, ApJ, 684, 29
Yi, S. K. 2008, in Hot Subdwarf Stars and Related Objects ASP Conf. Ser. Vol. 392, Edited by Ulrich Heber, C. Simon Jeffery, and Ralf Napiwotzki., p.3
Yi, S., Demarque, P., & Oemler, A., Jr. 1997, ApJ, 486, 201
Yi, S., Demarque, P., & Oemler, A., Jr. 1998, ApJ, 492, 480
Yi, S. K., Lee, J., Sheen, Y.-K., Jeong, H., Suh, H., & Oh, K. 2011, preprint (arXiv1107.0005)
Weidemann, V. 2000, A&A, 363, 647

TABLE 1
MEANING OF ACRONYMS

Acronyms	Meanings
CDS	Cluster disintegration scenario
SC	Star cluster
GC	Globular cluster
HNS	Helium-normal stars
HRS	Helium-rich stars
MS	Main-sequence (stars)
MGC	Massive GCs (i.e., SCs with HRSs)
GMCMF	GMC mass function

TABLE 2
MODEL PARAMETERS AND THEIR PHYSICAL MEANINGS

Parameters	Physical meanings
α_1	The IMF slope for HNSs
α_2	The IMF slope for HRSs
$m_l, 1$	The lower mass cut-off of the IMF for HNSs
$m_u, 1$	The upper mass cut-off of the IMF for HNSs
$m_l, 2$	The lower mass cut-off of the IMF for HRSs
$m_u, 2$	The upper mass cut-off of the IMF for HRSs
β	The slope of the GMC mass function (GMCMF)
$M_{\rm thres}$	The threshold mass for MGC formation
$\epsilon_{\rm bsc}$	The formation efficiency of bound SCs within GMCs (fixed)

TABLE 3
THE DEFINITION OF PHYSICAL QUANTITIES INVESTIGATED IN THE PRESENT STUDY

Quantity	Meaning
$f_{\rm m,He}$	The mass fraction of fresh helium gas in a SC
$f_{\rm m,agb}$	The mass fraction of AGB ejecta in a SC
$f_{\rm He}$	The mass fraction of MS HRSs in a SC
$F_{\rm n,mgc}$	The number fraction of MGCs with HRSs in all genuine GCs
$F_{\rm m,mgc}$	The mass fraction of MGCs with HRSs in all genuine GCs
$F_{\rm t,He}$	The mass fraction of HRSs in a spheroid
$F_{\rm He}$	The mass fraction of MS HRSs in a spheroid

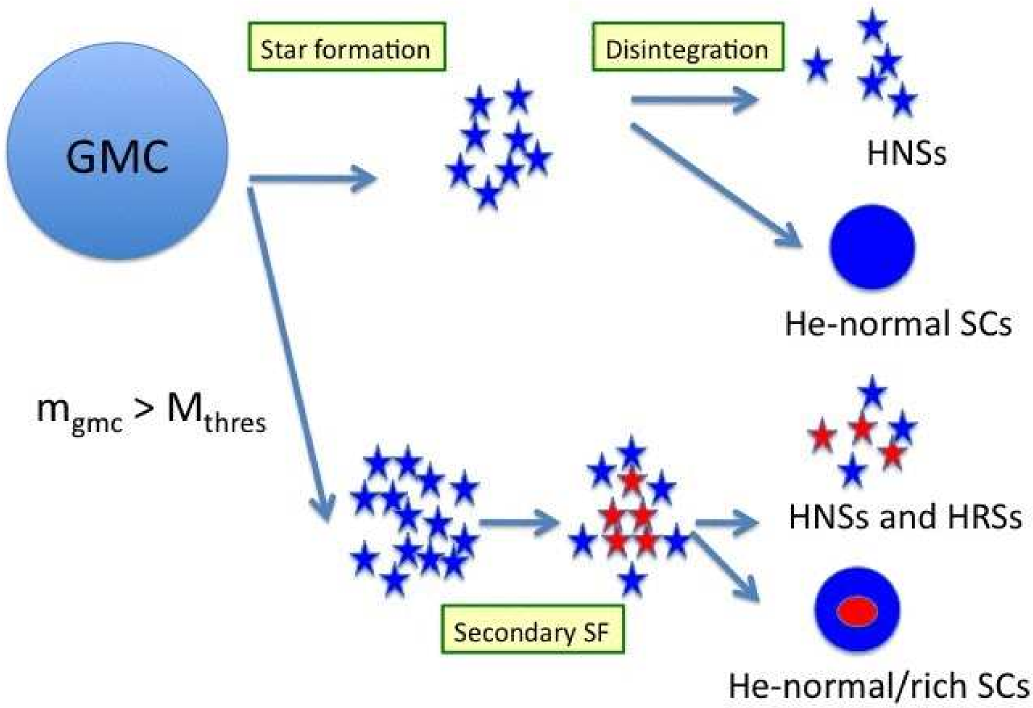

Fig. 1.— An illustration for the whole picture of the star formation processes from GMCs in the CDS. The evolution of newly formed HNSs (blue stars) depends on the masses of their host GMCs ($m_{\rm gmc}$). HNSs formed in lower mass GMCs with $m_{\rm gmc} \leq M_{\rm thres}$ can not form HRSs (red stars) from gaseous ejecta of their AGB stars owing to the shallow gravitational potentials of the SCs. The HNSs can finally evolve either field stars after disintegration of their host SCs or into bound SCs with only HNSs. On the other hands, if $m_{\rm gmc} > M_{\rm thres}$, then gaseous ejecta of AGB stars among HNSs can be converted into new HRSs (red stars) owing to deeper gravitational potentials of the SCs. The stars in the SCs can become either field HNSs/HRSs or main components of massive SCs with HRSs depending on physical properties of the SCs (i.e., on whether the SCs become disintegrated). The field HRSs in galactic spheroids are responsible for the UV upturn in the CDS.

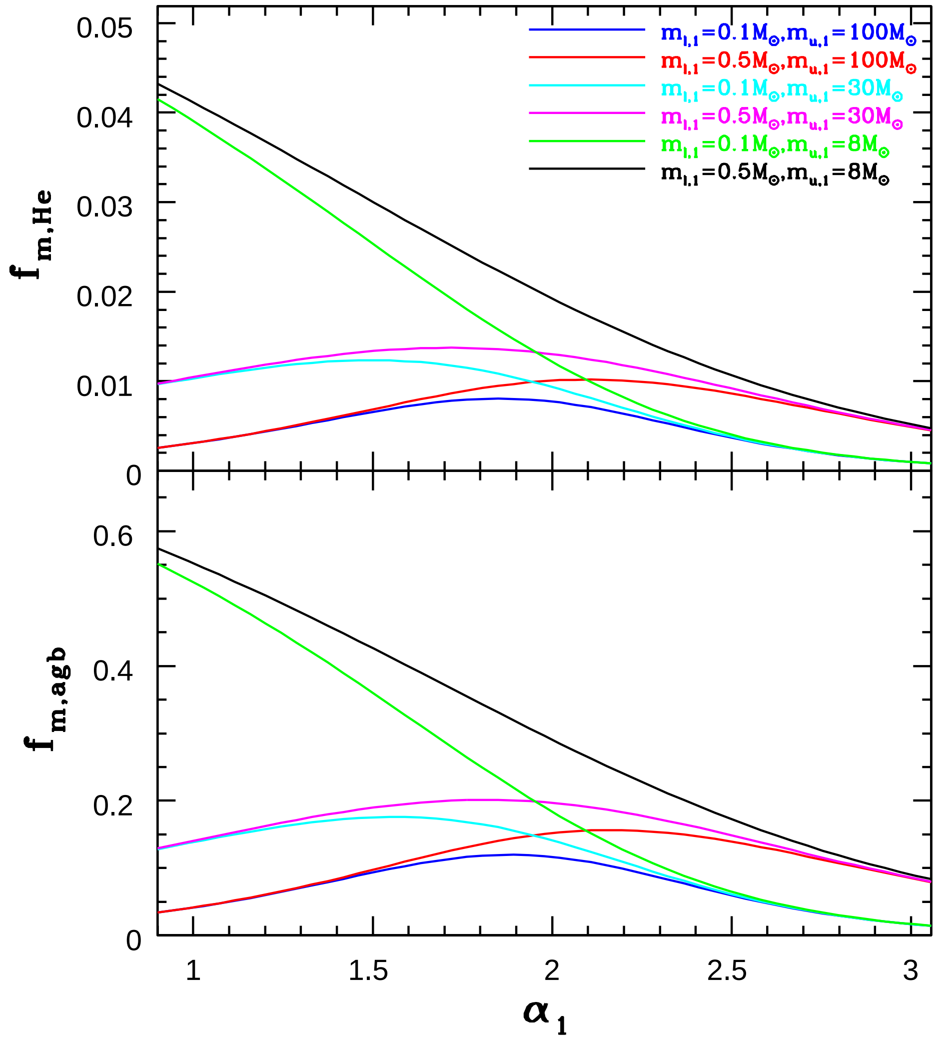

FIG. 2.— The dependences of the mass fractions of fresh helium gas ($f_{\rm m,He}$; upper) and those of the AGB ejecta ($f_{\rm m,agb}$; lower) in SCs on the IMF slopes of SCs (α_1) for different lower ($m_{\rm l,1}$) and upper ($m_{\rm u,1}$) mass cut-offs: $m_{\rm l,1} = 0.1{\rm M}_{\odot}$ and $m_{\rm u,1} = 100{\rm M}_{\odot}$ (blue), $m_{\rm l,1} = 0.5{\rm M}_{\odot}$ and $m_{\rm u,1} = 100{\rm M}_{\odot}$ (red), $m_{\rm l,1} = 0.1{\rm M}_{\odot}$ and $m_{\rm u,1} = 30{\rm M}_{\odot}$ (cyan), $m_{\rm l,1} = 0.5{\rm M}_{\odot}$ and $m_{\rm u,1} = 30{\rm M}_{\odot}$ (magenta), $m_{\rm l,1} = 0.1{\rm M}_{\odot}$ and $m_{\rm u,1} = 8{\rm M}_{\odot}$ (green), and $m_{\rm l,1} = 0.5{\rm M}_{\odot}$ and $m_{\rm u,1} = 8{\rm M}_{\odot}$ (black). Here $\alpha_1 = 2.35$ corresponds to the Salpeter IMF.

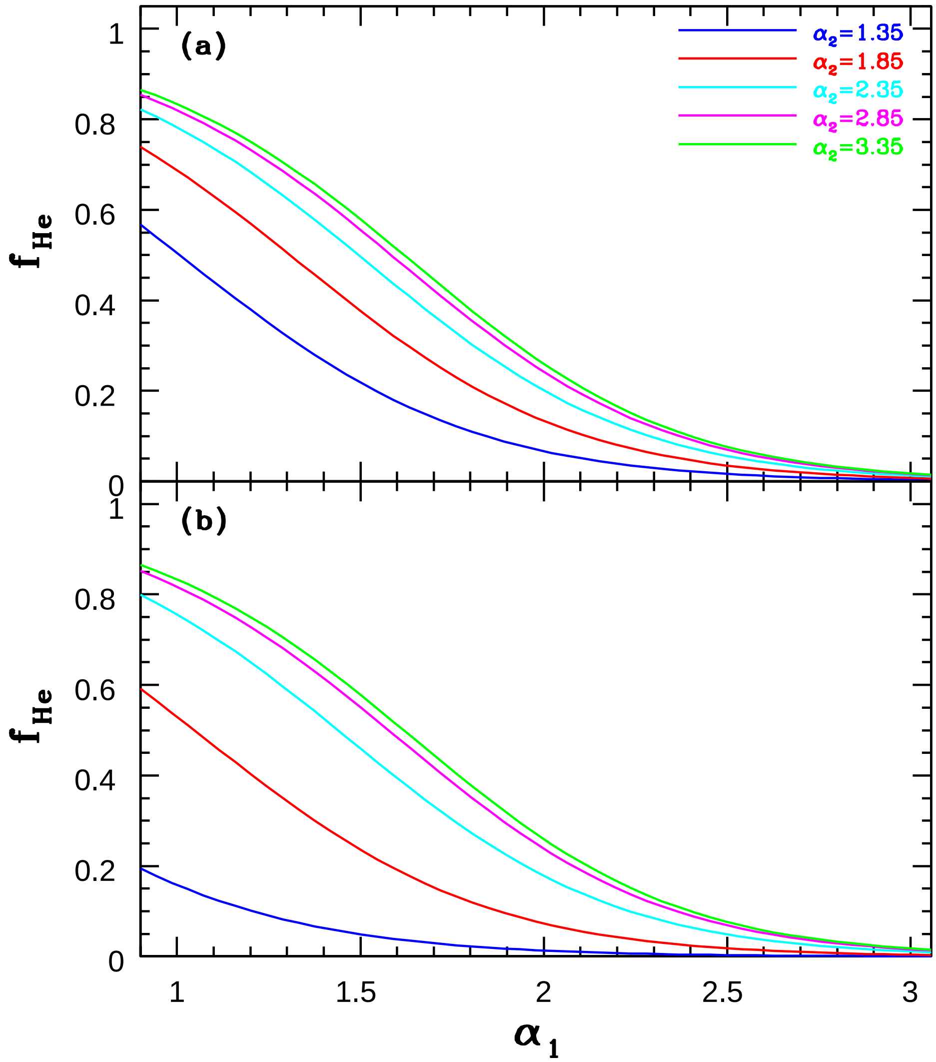

FIG. 3.— The dependences of mass fractions of MS HRSs ($f_{\rm He}$) on α_1 in SCs for $m_{\rm l,2} = 0.1{\rm M}_\odot$ and $m_{\rm u,2} = 8{\rm M}_\odot$ (upper), and $m_{\rm l,2} = 0.1{\rm M}_\odot$ and $m_{\rm u,2} = 100{\rm M}_\odot$ (lower) for different IMF slopes for HRSs (α_2): $\alpha_2 = 1.35$ (blue), $\alpha_2 = 1.85$ (red), $\alpha_2 = 2.35$ (cyan), $\alpha_2 = 2.85$ (magenta), and $\alpha_2 = 3.35$ (green). Here $\alpha_1 = 2.35$ corresponds to the Salpeter IMF.

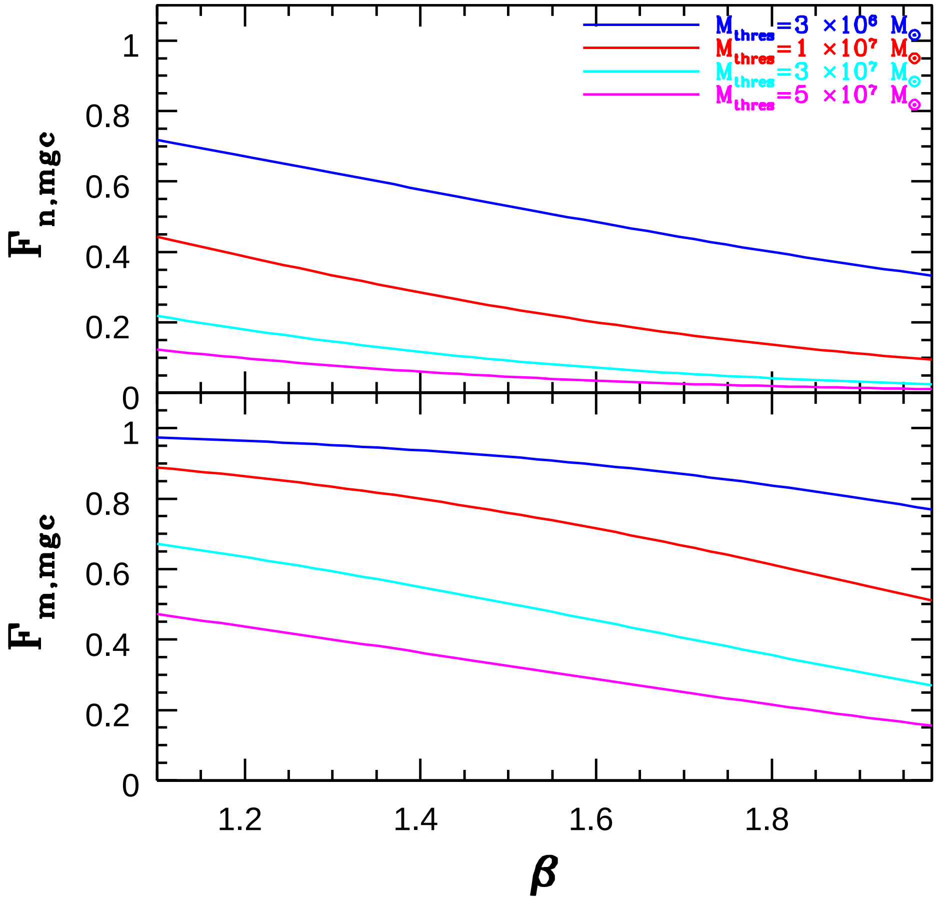

FIG. 4.— The dependences of number and mass fractions of massive GCs with HRSs ($F_{\rm n,mgc}$ and $F_{\rm m,mgc}$, respectively) in the Galactic "genuine" GCs (formed from GMCs with $m_{\rm gmc} \geq 10^6 {\rm M}_{\odot}$) on GMCMF slopes ($\beta$) for different threshold masses for MGC formation ($M_{\rm thres}$): $M_{\rm thres} = 3 \times 10^6 {\rm M}_{\odot}$ (blue), $M_{\rm thres} = 1 \times 10^7 {\rm M}_{\odot}$ (red), $M_{\rm thres} = 3 \times 10^7 {\rm M}_{\odot}$ (cyan), and $M_{\rm thres} = 5 \times 10^7 {\rm M}_{\odot}$ (magenta).

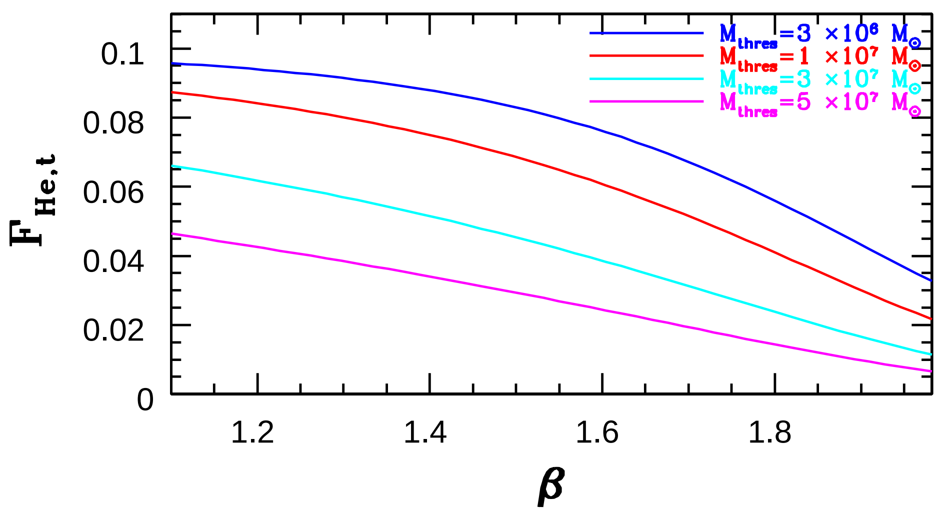

FIG. 5.— The dependences of the mass fractions of HRSs ($F_{\rm He,t}$) in galactic spheroids on GMCMF slopes (β) for canonical IMFs ($\alpha_1 = 2.35$) and different threshold masses for MGC formation ($M_{\rm thres}$): $M_{\rm thres} = 3 \times 10^6 {\rm M}_{\odot}$ (blue), $M_{\rm thres} = 1 \times 10^7 {\rm M}_{\odot}$ (red), $M_{\rm thres} = 3 \times 10^7 {\rm M}_{\odot}$ (cyan), and $M_{\rm thres} = 5 \times 10^7 {\rm M}_{\odot}$ (magenta).

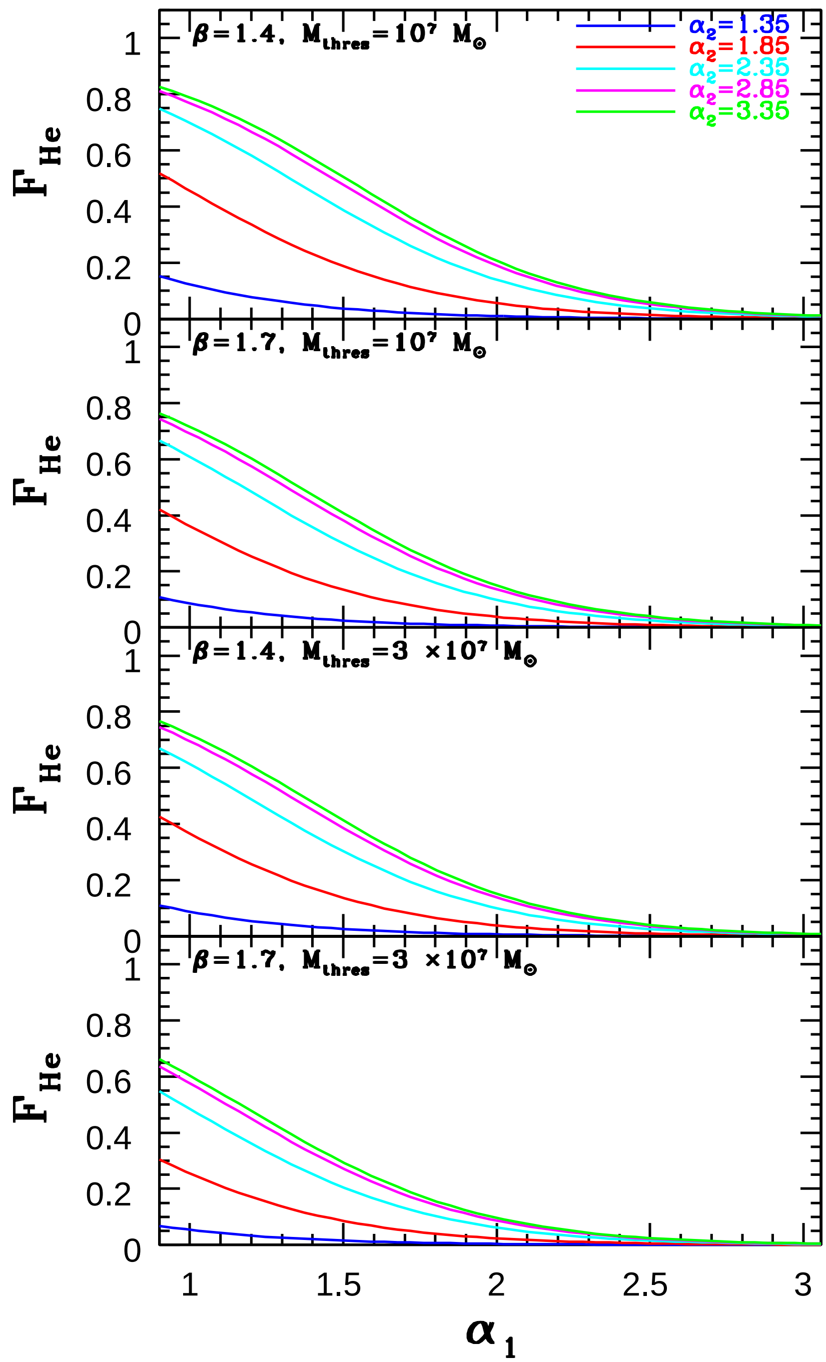

FIG. 6.— The dependences of mass fractions of MS HRSs in galactic spheroids ($F_{\rm He}$) on α_1 in the models with $\beta = 1.4$ and $M_{\rm thres} = 10^7 \rm M_\odot$ (top), $\beta = 1.7$ and $M_{\rm thres} = 10^7 \rm M_\odot$ (the second from top), $\beta = 1.4$ and $M_{\rm thres} = 3 \times 10^7 \rm M_\odot$ (the second from bottom), $\beta = 1.7$ and $M_{\rm thres} = 3 \times 10^7 \rm M_\odot$ (bottom) for different α_2: $\alpha_2 = 1.35$ (blue), $\alpha_2 = 1.85$ (red), $\alpha_2 = 2.35$ (cyan), $\alpha_2 = 2.85$ (magenta), and $\alpha_2 = 3.35$ (green).

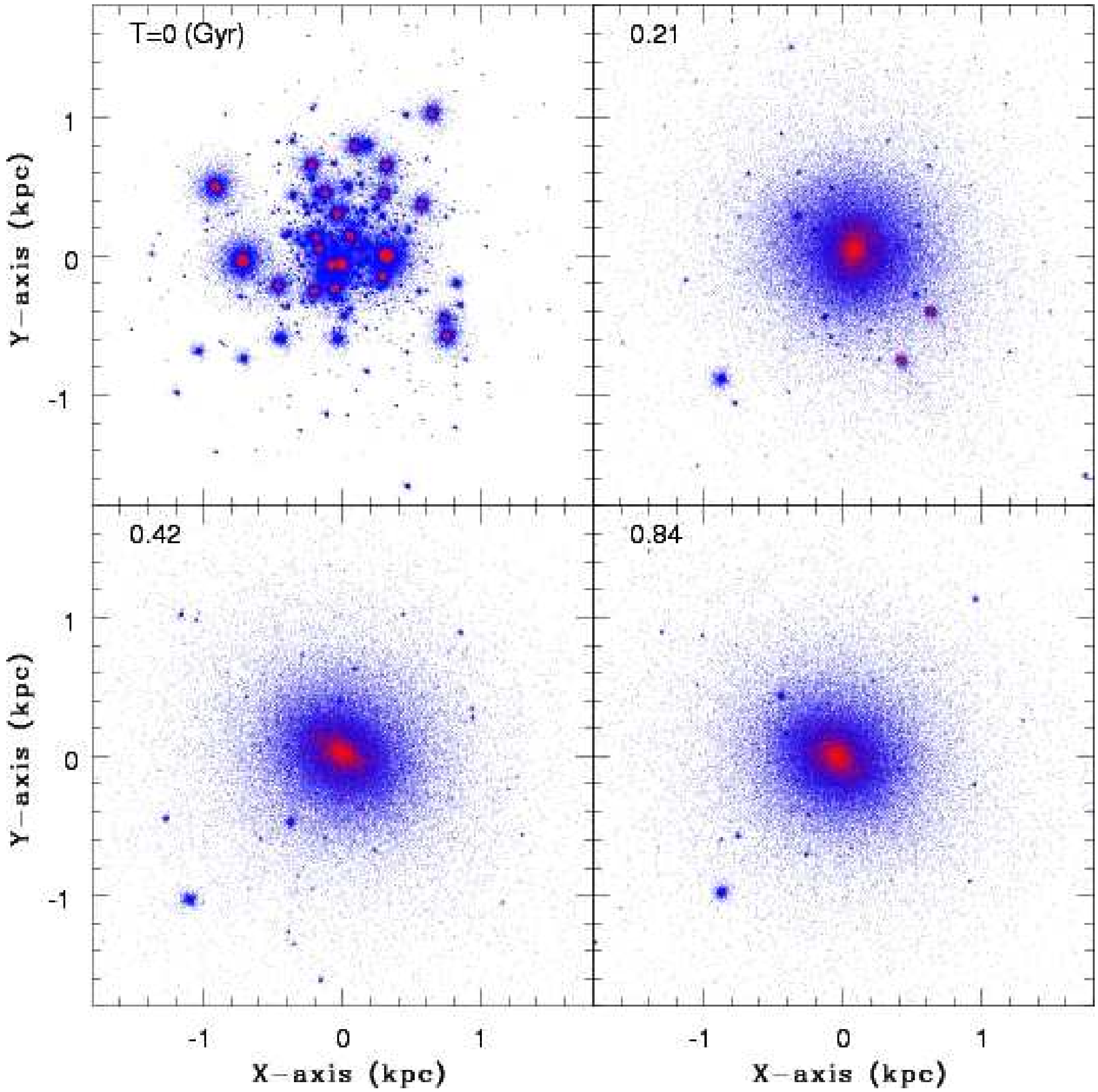

FIG. 7.— Time evolution of the distributions of HNSs (blue) and HRSs (red) projected onto the x-y plane for the standard model. The time (T in units of Gyr) that has elapsed since the simulation starts is shown in the upper left corner of each panel.

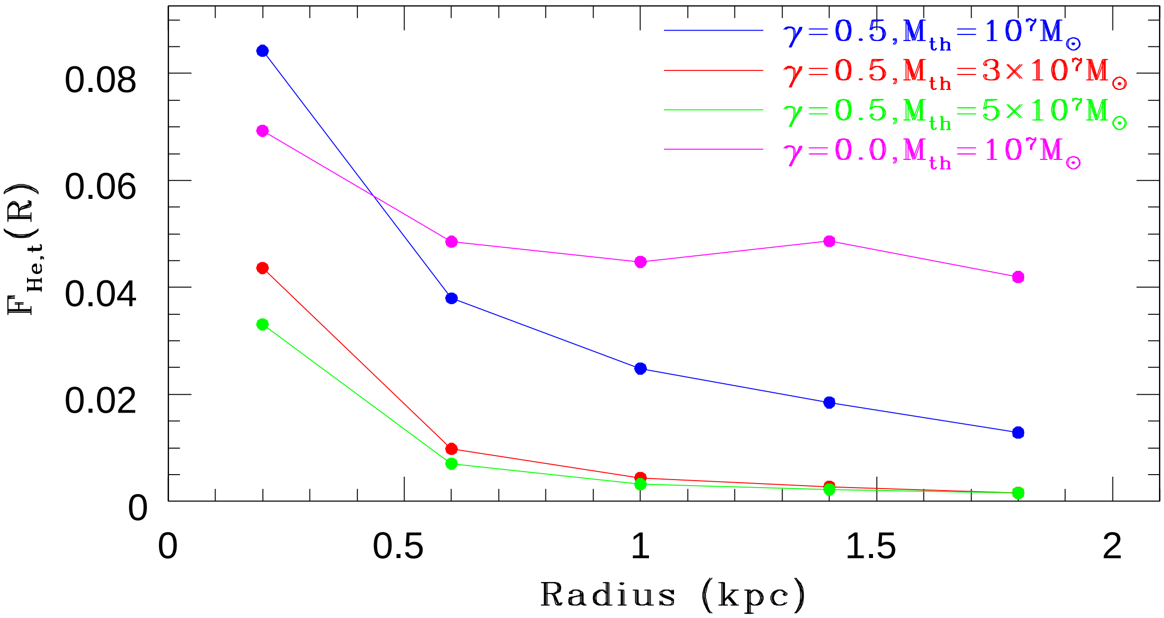

FIG. 8.— The radial gradients of mass fractions of HRSs ($F_{\mathrm{He,t}}$) in models with different parameters: $\gamma = 0.5$ and $M_{\mathrm{thres}} = 10^7 \mathrm{M}_\odot$ (blue), $\gamma = 0.5$ and $M_{\mathrm{thres}} = 3 \times 10^7 \mathrm{M}_\odot$ (red), $\gamma = 0.5$ and $M_{\mathrm{thres}} = 5 \times 10^7 \mathrm{M}_\odot$ (green), and $\gamma = 0.0$ and $M_{\mathrm{thres}} = 10^7 \mathrm{M}_\odot$ (magenta).